# Recent Advances in Proof Systems for Modal Logic

Sara Negri

*Department of Philosophy*
*P.O.Box 24 (Unioninkatu 40 A),*
*00014 University of Helsinki, Finland*

In recent years, a number of challenges has been faced in the proof theory of modal and related logics, especially in relation to their applications in the widening field of philosophical logic. However, as discussed in [6], uniform methods for generating analytic calculi for these logics have been developed in full generality only for normal modal logics with geometric frame conditions. On the one hand, frame conditions beyond geometric implications are common in multimodal logics such as systems used in formal epistemology [4] and in intermediate logics [1]. On the other hand, a relaxation of the framework of normal modal logics imposed by Kripke semantics is essential for avoiding the presuppositions of a logically omniscient and perfect reasoner and for capturing a non-monotonic notion of conditionals. The talk will focus on the extension of the methodology of labelled sequent calculi in the aforementioned two directions.

First, the method of extension of sequent calculi with rules will be generalized to allow for *systems of rules* [8] to cover the class of generalised geometric implications, a class that includes the frame properties that correspond to formulas in the Sahlqvist fragment. Further, it will be shown how to turn arbitrary first-order frame conditions into a finite collection of geometric rules, through an appropriate conservative semidefinitional extension of the language. In particular, this "geometrization of first order logic" yields complete sequent calculi for all modal logics defined by first-order frame conditions [1].

Second, a revision of the standard relational semantics in the direction of a general topological semantics will be shown amenable of a formal proof-theoretical treatment, in particular through an analysis of Lewis conditional logic as a labelled sequent calculus based on similarity relations and indexed modalities [10]. At the same time, the various stages of the methodology will be illustrated: a process of abduction to obtain the specification of the semantic framework; the use of the guidelines of proof-theoretic semantics to obtain calculi with good structural properties [9]; a direct Tait-Schütte-Takeuti style completeness proof [5]; and finally, a syntactic counterpart of semantic filtration to extract finite countermodels and thereby to turn non-constructive completeness proofs into effective decision procedures [2,7].

## References

[1] Dyckhoff, R. and Negri, S. Proof analysis in intermediate logics. *Archive for Mathematical Logic*, vol. 51, pp. 71–92, 2012.
[1] Dyckhoff, R. and Negri, S. Geometrization of first-order logic, ms., 2014.
[2] Garg, D., Genovese, V. and Negri, S. Countermodels from Sequent Calculi in Multi-Modal Logics. *Proceedings of LICS 2012*, IEEE Computer Society, pp. 315-324, 2012.
[3] Lewis, D. *Counterfactuals*. Blackwell, 1973.
[4] Maffezioli, P., Naibo, A. and Negri, S. The Church-Fitch knowability paradox in the light of structural proof theory. *Synthese*, vol. 190, pp. 2677-2716, 2013.
[5] Negri, S. Kripke completeness revisited. In G. Primiero and S. Rahman (eds), *Acts of Knowledge - History, Philosophy and Logic*, College Publications, 2009.
[6] Negri, S. Proof theory for modal logic. *Philosophy Compass*, vol. 6, pp. 523-538, 2011.
[7] Negri, S. Proofs and countermodels in non-classical logics. *Logica Universalis*, vol. 8, pp. 25-60, 2014
[8] Negri, S. Proof analysis beyond geometric theories: from rule systems to systems of rules. *Journal of Logic and Computation*, in press.
[9] Negri, S. and von Plato, J. Meaning in use. In H. Wansing (ed), *Dag Prawitz on Proofs and Meaning*, Trends in Logic, Springer, in press.
[10] Negri, S. and Sbardolini, G. Proof analysis for Lewis counterfactuals, ms., 2014.