The Existence of Proof Systems

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During the last hundred years proof systems of all kinds have been developed for a great variety of logics. These proof systems can often be used to establish that the corresponding logics have nice properties, such as decidability, interpolation or Skolemization. Results stating that a logic does not have certain proof systems are less common. In this talk a method is introduced to prove such negative results. The method establishes a connection between the existence of certain proof systems for a logic and certain regular properties that the logic satisfies. The talk focusses on (intuitionistic) modal logics, although the method is applicable to other logics as well. The regular properties considered in this talk are variants of interpolation, and the developed method can be used not only to obtain the negative results, but also to prove uniform interpolation for several classical and intuitionistic modal logics. The method is in fact inspired by the syntactic proof that intuitionistic logic has uniform interpolation by Pitts.

The method makes use of sequent calculi, but in a very abstract form. The key property of rules that this method uses is that of being *focussed*, a property that expresses the structurality of a rule. Many of the standard sequent rules for connectives have this property and thus are focussed. In [2] it is shown that if a modal logic has a proof system that consists of focussed rules, then it has uniform interpolation, which implies that the many modal logics without uniform interpolation [1,3] cannot have focussed proof systems. The generality of the notions involved makes the method applicable to many other logics, for example to intermediate logics.

In how far other proof systems lend themselves to this approach is still not clear. Besides the technical results above, such unresolved issues as well as related conjectures will be addressed during the talk.

References

[1] Ghilardi, S. and M. Zawadowski, “Sheaves, Games, and Model Completions: A Categorical Approach to Nonclassical Propositional Logics,” Trends in Logic (Book 14), Springer, 2002.

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[3] Maksimova, L., *Craig’s theorem in superintuitionistic logics and amalgamable varieties of pseudo-Boolean algebras*, Algebra Logika **16** (1977), pp. 643–681.