A Simple Semantics for Aristotelian Apodeictic Syllogistics

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Abstract

We give a simple definition of validity for syllogisms involving necessary and assertoric premises which validates all and only the Aristotelian apodeictic syllogisms.

Keywords: Aristotle, modal syllogistic, semantics, two Barbaras

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1 The problem

The first systematic study of reasoning and inference in the West was done by Aristotle. However, while his assertoric theory of syllogistic reasoning is provably sound and complete for the class of models validating the inferences in the traditional square of opposition [5, p. 100], his modal syllogistic, developed in chapters 3 and 8–22 of the *Prior Analytics* [1], has the rather dubious honor of being one of the most difficult to understand logical systems in history. Starting with some of his own students, many have considered Aristotle's modal syllogistic to be anywhere from confused to simply wrong [7, ch. 1]. In support of these claims, many critics point to what is called the "two Barbaras problem", that is, Aristotle's treatment of syllogisms of the form LXL Barbara and XLL Barbara.[1] According to Aristotle, arguments of the form

Necessarily A belongs to all B.
B belongs to all C.
Therefore, necessarily A belongs to all C.

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[1] See §2 for an explanation of the notation. Throughout this paper we make use of the traditional medieval mnemonic names of syllogisms [13, p. 21].

are valid, while arguments of the form

A belongs to all B.
Necessarily B belongs to all C.
Therefore, necessarily A belongs to all C.

are invalid [1, 30a15–30a33]. Many people have found this position to be inconsistent. [2] Aristotle's student Theophrastus argued that both syllogisms are invalid [7, p. 15], since nothing should follow when one premise is necessary and the other assertoric. Łukasiewicz, whose views on Aristotle's modal syllogistic [5] have been extremely influential on modern approaches to the system, has argued that both syllogisms are valid [7, p. 15]. Łukasiewicz says that "Aristotle's modal syllogistic is almost incomprehensible because of its many faults and inconsistencies" and "modern logicians have not as yet been able to construct a universally acceptable system of modal logic which would yield a solid basis for... Aristotle's work" [5, p. 132]. One of the "faults and inconsistencies" is Aristotle's acceptance of LXL Barbara while rejecting of XLL Barbara. Later attempts have been made to give a consistent interpretation of Aristotle's modal syllogistic. McCall [7] gave a syntactic theory which coincides exactly with the apodeictic fragment of the Aristotelian theory (the fragment containing just the necessity and assertoric modal operators). More recently, Johnson [3,4], Thomason [15], and Malink [6] have given semantics corresponding to McCall's syntax, showing that Aristotle's apodeictic fragment is consistent, if, given the complexity of their semantic models, rather unintuitive.

We offer a new approach to the apodeictic fragment of Aristotelian syllogistics, which provides a clear and simple definition of validity that validates all and only those apodeictic syllogisms accepted by Aristotle. First, in §2 we define the notation we use in this paper. Previous attempts at giving syntactic and semantic characterization of the modal syllogistic are considered in §§4,5. The definition of validity that we give provides a formalization of the philosophical interpretation of Aristotle's apodeictic syllogistic given by Rescher in [8], and refined by McCall in [7]; we discuss this interpretation in §3, and then give our new formalism in §6. In §7 we show it is adequate for the pure necessary/assertoric fragment, and discuss the problems we have faced extending this formalism to the fragment which also contains the possibility operator. We conclude with some comments about future work in §8.

2 Notation

Syllogistics is a term logic, so we fix a set TERM of basic terms, and let capital letters $A, B, C \ldots$ range over TERM. (Assertoric) categorical propositions are formed from

[2] And some people disagree that there is even a separate *modal* syllogistics at all [10].

1st	2nd	3rd
A — B	B — A	A — B
B — C	B — C	C — B
A — C	A — C	A — C

Figure 1. The Three Figures

copulae a, e, i, o and terms as follows:

AaB	'A belongs to all B'	$\leftrightarrow$ 'All B are A'	(universal affirmative)
AeB	'A belongs to no B'	$\leftrightarrow$ 'No B is A'	(universal negative)
AiB	'A belongs to some B'	$\leftrightarrow$ 'Some B is A'	(particular affirmative)
AoB	'A does not belong to some B'	$\leftrightarrow$ 'Some B is not A'	(particular negative)

The term preceding the copula is called the predicate term and the term succeeding it is called the subject term. We follow McCall and use L, X, and M to denote the necessary, assertoric, and possible modes, respectively. [3] Hence, if φ is an assertoric categorical proposition, $L\varphi$, $X\varphi$, and $M\varphi$ are modal categorical propositions. (Note that the "assertoric" mode is not any different from the ordinary propositional mode. We will often designate assertoric propositions without the X.) Categorical propositions, both assertoric and modal, can be combined to form syllogisms.

Definition 2.1 A triple $\mathcal{S} = \langle M, m, c \rangle$, where M, m, and c are categorical propositions, is a *syllogism* if M, m, and c contain exactly three distinct terms, of which the predicate of c (called the major term) appears in M and the subject of c (called the minor term) appears in m, and M and m share a term (called the middle term) which is not present in c.

We call M the major premise, m the minor, and c the conclusion. The three ways that major, minor, and middle terms in the premises can be arranged are called figures (see Figure 1). A figure with three copulae added is called a 'mood'; by, e.g., 'LLL Barbara' we mean the mood Barbara with each of the premises prefaced with mode L.

3 Rescher's interpretation

A supposed drawback of Aristotle's modal syllogistic according to Łukasiewicz is that it "does not have any useful application to scientific problems" [5, p. 181]. In contrast with this conclusion, Rescher believes not only that the modal syllogistic can be given a consistent interpretation, but that, in fact, this interpretation is based on Aristotle's theory of scientific knowledge and inference. Rescher describes attempts such those of Łukasiewicz and Becker [2] as "blind alleys, as regards the possibility of interpreting Aristotle's discussion as it stands, without introducing numerous 'corrections'" [8,

[3] We omit from discussion the mode Q 'contingent'.

p. 165]. He argues that the problem of these formalisms was that they force an incorrect interpretation of the *Prior Analytics*. To address this, Rescher develops a non-formal account of the *Prior Analytics* which stresses the scientific nature of the various modal deductions. He argues that:

> The key to Aristotle's theory lies, I am convinced, in viewing the theory of modal syllogisms of the *Analytica Priora* in the light of the theory of scientific reasoning of the *Analytica Posteriora* [8, p. 170].

On his analysis, the major premise is treated as a general scientific principle or rule and the minor premise as a specific instance of the general rule [8, p. 171]. Further,

> [A] rule that is necessarily (say) applicable to all of a group will be necessarily applicable to any sub-group, pretty much regardless of how this sub-group is constituted. On this view, the necessary properties of a genus must necessarily characterize even a contingently differentiated species. If all elms are necessarily deciduous, and all trees in my yard are elms, then all trees in my yard are necessarily deciduous (even though it is not necessary that the trees in my yard be elms) [8, p. 172].

This interpretation allows him to make a principled distinction between LXL Barbara and XLL Barbara, since in the first case, the general rule is necessary, and the particular instance falls under that necessary rule. The conclusion that results should then be necessary. However, if the general rule is only assertoric, then the conclusion shouldn't be necessary, since for Aristotle, the assertoric generally does not entail the necessary.

McCall rightly points out that this interpretation only works for the first-figure syllogisms with mixed necessary and assertoric premises. In the case of second and third figure syllogisms, such as XLL Camestres, the minor premise is the general rule, and the major premise is the special case. Further, attempting to reduce the validity of these other figures to that of the first figure is problematic, not least because one would have to justify the conversion rules used in the reduction. As an alternative, drawing inspiration from the medieval doctrine of distribution, McCall points out that, with two exceptions, the general rule is the premise in which the middle term is distributed, and in a valid syllogism the special case can be "upgraded" to the modality of the general rule. A term is distributed in a proposition if "it *actually* denotes or refers to, in that premiss, the whole of the class of entities which it is *capable* of denoting" [7, p. 25]. In AaB, B is distributed; in AeB, both terms are distributed; in AiB, neither term is distributed; in AoB, A is distributed. The two restrictions are the following: (1) general rules cannot be particular and (2) special cases cannot be negative [7, p. 26]. The first exception allows us to rule out XLL Baroco while the second exception allows us to avoid XLL Felapton and XLL Bocardo, which are not accepted as valid by Aristotle [1, 31a1–31a18, 31a14–31a33].

The models that we introduce in §6 take seriously this suggestion of Rescher that we understand modal syllogisms as making a statement about the relationship between a general scientific law and a special case falling under that law. We will give a precise definition of what counts as a special case, and make explicit how to "upgrade" the modality of the special case to that the general rule. Thus, we will be able to show that if we accept Rescher's interpretation of the modal syllogistic, a consistent theory of syllogistic reasoning can be extracted from Aristotle's works.

4 Syntactic characterizations of the apodeictic fragment

McCall then used his "completion" (as he calls it) of Rescher's interpretation as the basis for developing a syntactic system characterizing Aristotle's apodeictic fragment of the syllogistic. It is based on the rules of conversion and the perfect syllogisms that Aristotle defined for the apodeictic syllogistic in the *Prior Analytics*. McCall shows that from propositional logic plus an axiomatization of the assertoric syllogistic supplemented with six modal axioms, and four laws of modal conversion and subordination, it is possible to deduce all of the valid apodeictic syllogisms and reject all of the ones that are invalid according to Aristotle [7, §14]. The six modal axioms are LXL Barbara, LXL Cesare, LXL Darii, LXL Ferio, LLL Baroco, LLL Bocardo, and the conversion and subordination rules are:

- from $LAiB$ infer $LBiA$
- from $LAaB$ infer AaB
- from $LAiB$ infer AiB
- from $LAoB$ infer AoB

McCall made no attempt to give a semantic grounding for his syntactic theory.

Rescher, along with Parks, later developed his interpretation into a proof-theoretic account which simplifies McCall's approach [9], but which only deals with the L-X fragment (whereas McCall's syntactic theory can be extended to the L-X-M fragment). At the heart of their account is the following observation:

> The leading idea of our proposal is that given syllogistic terms α and β it is possible to define yet another term $[\alpha\beta]$ to represent the β-species of α... they are those α's which must be β's relative to the hypothesis that they are α's (by conditional or relative necessity) [9, p. 678–679].

This idea is based on Aristotle's notion of ekthesis, which allows for deriving universal propositions from particular ones, and which Aristotle uses to give proofs of the oblique moods LLL Baroco and LLL Bocardo [9, §3]. (For more information on ekthesis and its role in Aristotelian syllogistic proofs, see [12]). This observation allows us to move from "A belongs to all B" to "all Bs, given that they are As, are necessarily A, with relative necessity, given that they are in fact Bs." This notion of relative necessity plays a key role in development of Rescher and Park's system, which has just four conversion rules together with the perfect assertoric and wholly apodeictic syllogisms as axioms. The four conversion rules are as follows:

$$\vdash AaB \Rightarrow \vdash L[BA]aB$$
$$\vdash AiB \Rightarrow \vdash L[BA]iB$$

$$\vdash LAaB \Rightarrow \vdash LAa[CB]$$

$$\vdash LAeB \Rightarrow \vdash LAe[CB]$$

The complex term $[AB]$ is read 'A-conditioned-by-B' or 'A's which are B'. These rules can be understood as follows:

- If A belongs to all B, then being B's which are A necessarily belongs to all B.
- If A belongs to some B, then being a B which is A necessarily belongs to some B.
- If A necessarily belongs to all B, then A necessarily belongs to all those C which are B.
- If A necessarily does not belong to any B, then A necessarily belongs to no C which is a B.

Rescher and Parks prove the consistency of their theory only in an indirect fashion (by reducing the apodeictic syllogistic to the assertoric one, which was proved consistent in [11]).

5 Previous semantic attempts

Later authors have attempted to build semantics for McCall's or an equivalent axiomatization; three rigorous approaches are those of Johnson [3], Thomason [15], and [6]. While these semantics are adequate in so far as they validate all of McCall's (and hence Aristotle's) theses, and reject those that should be rejected, they are not very appealing on grounds of both aesthetics and explanatory value. The systems are very complicated and could be labeled *ad hoc* because they are not motivated beyond being adequate to characterize (McCall's version) of Aristotle's theory.

5.1 Johnson's model

The semantics given by Johnson in [3] are adequate to prove the completeness of the apodeictic fragment of McCall's formalization.

Definition 5.1 A *Johnson-syllogistic model* is a quintuple

$$\mathfrak{M}^{\mathrm{J}} = \langle W, V^e, V^a, V^e_c, V^a_c \rangle,$$

where W is a set and the V^i_j are functions from TERM to 2^W meeting the following conditions:

(i) $V(A) := V^e(A) \cup V^a(A)$

(ii) $V^e(A) \neq \emptyset$

(iii) For each A, $V^j_k(A) \cap V^m_n(A) = \emptyset$ iff either $j \neq m$ or $k \neq n$; and for each A, $V^e(A) \cup V^a(A) \cup V^e_c(A) \cup V^a_c(A) = W$.

(iv) If $V(C) \subset V^e_c(B)$ and $V(A) \subset V(B)$ then $V(A) \subset V^e_c(C)$.

(v) If $V(B) \subset V^e(C)$ and $V(A) \cap V(B) \neq \emptyset$ then $V^e(A) \cap V^e(C) \neq \emptyset$.

(vi) If $V(B) \subset V^e_c(C)$ and $V(A) \cap V(B) \neq \emptyset$ then $V^e(A) \cap V^e_c(C) \neq \emptyset$.

(vii) If $V(C) \subset V^e(B)$ and $V^e(A) \cap V^e_c(B) \neq \emptyset$ then $V^e(A) \cap V^e_c(C) \neq \emptyset$.

We think of $V^e(A)$ as the set of things which are essentially A, $V^a(A)$ as the things which are accidentally A, $V^e_c(A)$ is the set of things essentially non-A, and $V^a_c(A)$ is the set of things accidentally non-A.

The truth conditions for categorical propositions are as expected:

Definition 5.2

$$\mathfrak{M}^{\mathrm{J}} \models AaB \quad \text{iff} \quad V(B) \subset V(A).^{4}$$

$$\mathfrak{M}^{\mathrm{J}} \models AiB \quad \text{iff} \quad V(B) \cap V(A) \neq \emptyset.$$

$$\mathfrak{M}^{\mathrm{J}} \models AeB \quad \text{iff} \quad \mathfrak{M}^{\mathrm{J}} \not\models AiB.$$

$$\mathfrak{M}^{\mathrm{J}} \models AoB \quad \text{iff} \quad \mathfrak{M}^{\mathrm{J}} \not\models AaB.$$

$$\mathfrak{M}^{\mathrm{J}} \models LAaB \quad \text{iff} \quad V(B) \subset V^e(A).$$

$$\mathfrak{M}^{\mathrm{J}} \models LAeB \quad \text{iff} \quad V(B) \subset V^e_c(A).$$

$$\mathfrak{M}^{\mathrm{J}} \models LAiB \quad \text{iff} \quad V^e(B) \cap V^e(A) \neq \emptyset.$$

$$\mathfrak{M}^{\mathrm{J}} \models LAoB \quad \text{iff} \quad V^e(B) \cap V^e_c(A) \neq \emptyset.$$

Thom criticizes these semantics in [14], and Johnson responded to Thom's objections in [4]. The revised system of [4] was intended to (a) allow that general terms may designate a property such that no object necessarily has this property (thus giving up (ii) above), (b) require that if some object has the property designated by a general term necessarily, then any object which has this property has it necessarily, and (c) be "intuitively graspable" [4, p. 171]. The system goes beyond Aristotelian modal logic by allowing singular sentences (that is, sentences involving constants instead of terms), but it is more restricted than McCall's syntax in that it does not account for M propositions. The semantics are substitutionally based. Thirteen conditions for an acceptable valuation function are given in §3, thus it is by no means clear that Johnson has succeeded with his goal (c) in the new semantics.

5.2 Thomason's models

Thomason feels that Johnson's semantics "is in some respects contrived" [15, p. 111], and offers a proposal of his own. Thomason finds fault with Johnson's semantics in that "the interpretations are explicitly required to satisfy Axioms 6–9 [LXL Cesare, Darii, and Ferio, and LLL Baroco] of L-X-M" [15, p. 112], and he introduces models which do away with this requirement.

[4] Note that this definition does not entail existential import, whereas Aristotle's definitions in the Square of Opposition do.

Definition 5.3 A *Thomason-syllogistic model* is a quintuple

$$\mathfrak{M}^{\mathrm{T}} = \langle W, \mathrm{Ext}, \mathrm{Ext}^{+}, \mathrm{Ext}^{-}, V \rangle,$$

where the Exts are functions assigning subsets of W to each term satisfying $\mathrm{Ext}^{+} \subseteq \mathrm{Ext}$, $\mathrm{Ext}^{+} \neq \emptyset$, $\mathrm{Ext}^{-} \cap \mathrm{Ext} = \emptyset$, and V is an ordinary two-valued valuation function.

The functions $\mathrm{Ext}(x)$, $\mathrm{Ext}^{+}(x)$ and $\mathrm{Ext}^{-}(x)$ should be understood as picking out that which is x, is x necessarily, and is necessarily not x respectively. The truth conditions for the assertoric propositions are the same as in Johnson's semantics (so they also do not satisfy existential import), while those for the modal propositions are defined as follows:

Definition 5.4

$$\begin{aligned}
V(LAaB) = T &\quad \text{iff} \quad \mathrm{Ext}(A) \subset \mathrm{Ext}^{+}(B) \\
V(LAeB) = T &\quad \text{iff} \quad \mathrm{Ext}(A) \subset \mathrm{Ext}^{-}(B) \\
V(LAiB) = T &\quad \text{iff} \quad \mathrm{Ext}^{+}(A) \cap \mathrm{Ext}^{+}(B) \neq \emptyset \\
V(LAoB) = T &\quad \text{iff} \quad \mathrm{Ext}^{+}(A) \cap \mathrm{Ext}^{-}(B) \neq \emptyset
\end{aligned}$$

Validity and consequence are defined on these models in the expected way. Then, the consequences of Axioms 6–9 on this class of models correspond exactly to the theorems of Johnson's axiomatization, which in turn corresponds exactly to Aristotle's theory [15, p. 120]. Since these models require the truth of Axioms 6–9 to be built into the interpretation function, Thomason does not find them adequate, and instead offers two further classes of models, which satisfy all the requirements previous outlined and additionally

(i) $\mathrm{Ext}(x) \cap \mathrm{Ext}(y) \neq \emptyset \Rightarrow \mathrm{Ext}(x) \cap \mathrm{Ext}^{+}(y) \neq \emptyset$

(ii) Both (i) and $\mathrm{Ext}(x) \subseteq \mathrm{Ext}^{-}(y) \Rightarrow \mathrm{Ext}(y) \subseteq \mathrm{Ext}^{-}(x)$ and $\mathrm{Ext}(x) \subseteq \mathrm{Ext}^{+}(y) \rightarrow \mathrm{Ext}^{-}(y) \subseteq \mathrm{Ext}^{-}(x)$.

Aristotle's theory of the apodeictic syllogistic coincides with the set of consequences of LLL Baroco and the conversion rule $LAeB \Rightarrow LBeA$ on the second class of models [15, p. 122] and with the set of validities of the third class of models [15, p.124]. Thus, if we build extra structure into the interpretation of the terms, we are able to recover Aristotelian syllogistics without further assumptions. However, it is not clear where the justification for this extra structure comes in, other than that its addition makes the system work. It would be preferable to have a justification which is less *ad hoc* and more grounded in Aristotelian philosophy.

5.3 Malink's models

A rather different approach is taken by Malink in [6]. Malink appeals to Aristotle's discussion of types of predication in the *Topics* for the philosophical grounding of his interpretation, and bases his reconstruction of the modal syllogistic on what he calls 'predicable-based modal copula' [6, p. 97]. In the *Topics*, there are four different types of predicables: genus with (a) *differentia*, (b) definition, (c) *proprium*, or (d) accident. These four types of predicables are based on two basic relations, essential predication ($\mathbf{E}ab$) and accidental predication (Υab). Malink characterizes the behavior of these two basic relations via the axiomatic system $\mathcal{A}$, consisting of seven definitions and five axioms, and which "is not intended to give an exhaustive description of Aristotelian predicable-semantics, but to capture only those aspects of it which are relevant for the formal proofs of modal syllogistic" [6, p. 98]. These definitions and axioms are interpreted in graphically-representable structures made up of the following elements:

$\bullet$	substance term, Σa
$\circ$	nonsubstance term, $\neg\Sigma a$
———	substantial essential predication, $\mathbf{E}ab$
......	merely accidental predication, Υab
− − −	non-substantial essential predication, $\tilde{\mathbf{E}}ab$

The predicative relations between terms are represented by downward paths in the diagrams, with the conventions that it is assumed that all substance terms are $\mathbf{E}$ predicated of themselves, and all nonsubstance terms are $\tilde{\mathbf{E}}$ predicated of themselves, and when both types of predication coincide, only $\mathbf{E}$ predication is drawn.

In Malink's system, assertoric, necessary, (merely) possible, and contingent categorical claims are formalized as follows:

$$XAaB \quad \Upsilon ab$$
$$XAeB \quad \forall z(\Upsilon bz \rightarrow \neg\Upsilon az)$$
$$XAiB \quad \exists z(\Upsilon bz \wedge \Upsilon az)$$
$$XAoB \quad \neg\Upsilon ab$$
$$LAaB \quad \widehat{\mathbf{E}}ab$$
$$LAoB \quad \mathbf{K}ab$$
$$LAiB \quad \exists z((\Upsilon bz \wedge \widehat{\mathbf{E}}az) \vee (\Upsilon az \wedge \widehat{\mathbf{E}}bz))$$
$$LAoB \quad \exists z(\Upsilon bz \wedge \mathbf{K}az) \vee \exists xv(\widehat{\mathbf{E}}bz \wedge \widehat{\mathbf{E}}av \wedge \forall u(\Upsilon au \wedge \widehat{\Sigma}u \rightarrow \mathbf{K}zu))$$
$$MAaB \quad \forall z(\Upsilon bz \rightarrow \bar{\Pi}az)$$
$$MAeB \quad \forall z(\Upsilon bz \rightarrow \neg\bar{\mathbf{E}}az) \wedge \forall z(\Upsilon \rightarrow \neg\bar{\mathbf{E}}bz)$$
$$MAiB \quad \exists z(\Upsilon bz \wedge \bar{\Pi}az)$$
$$MAoB \quad \neg\bar{\mathbf{E}}ab$$

$$QAa/eB\ \forall z(\Upsilon bz \to \Pi az)$$

$$QAi/oB\ \ \Pi ab$$

where $\Sigma a := \exists z\mathbf{E}za$, $\mathbf{K}ab := \Sigma a \wedge \Sigma b \wedge \neg\exists z(\Upsilon az \wedge \Upsilon bz)$, $\Pi ab := \neg(\Sigma a \wedge \Sigma b) \wedge \neg\mathbf{E}ab \wedge \neg\mathbf{E}ba \wedge ((\Sigma a \vee \Sigma b) \to \exists z(\Upsilon az \wedge \Upsilon bz))$, $\bar{\Pi}ab := \Pi ab \vee \Upsilon ab$, $\widehat{\mathbf{E}}ab := \mathbf{E}ab \vee \tilde{\mathbf{E}}ab$, $\widehat{\Sigma}ab := \exists z\mathbf{E}za$, $\bar{\mathbf{E}}ab := \mathbf{E}ab \vee (\Sigma a \wedge \Upsilon ab)$.

With this formalization, Malink is able to validate not only the apodeictic fragment but he can also makes sense of the merely possible and the contingent fragments, making his approach an improvement on both McCall's syntax as well as the models of Johnson and Thomason, which only work for the apodeictic fragments. However, this short overview of some of the aspects of Malink's reconstruction should be enough to demonstrate its extreme complexity, and there are other drawbacks with this approach which we discuss in the next section.

5.4 Discussion and critique

While these three types of models are semantically adequate in that their proofs are sound and their systems correspond to (a fragment of) Aristotelian syllogistic, there are a number of issues of their formalisms that we want to highlight. First, the semantics do not really explain what is going on in Aristotle's system. Each of the models introduces a primitive distinction between essential and nonessential predications. In Johnson, the quadripartite interpretation functions correspond to the notions of necessarily belonging, contingently belonging, contingently not belonging and necessarily not belonging. Thomason simplifies this to the tripartite distinction between what is necessary, what is necessarily not, and what is neither. Malink reduces this one more step, and distinguishes essential predication and accidental predication. While building these distinctions into the truth conditions and/or syntax is entirely adequate to capture Aristotle's notion of necessity and contingency, doing so reduces what explanatory power the models might have otherwise had.

Furthermore, none of the authors discussed how their semantics correlate with the or make sense of the new interpretation of Aristotle given by Rescher and discussed by McCall. Given that Rescher's interpretation gives a philosophical grounding for why Aristotle's modal syllogistic validates the syllogisms that it does, it is unfortunate that when Johnson, Thomason, and Malink develop their semantics, none of them discuss this philosophical grounding.

6 A new approach

Our new approach to the apodeictic syllogistic is based on making formal the "upgrade" criterion that McCall gives. Our models are standard models for quantified modal logic:

Definition 6.1 A *simple syllogistic model* is a tuple $\mathfrak{M}^{\mathrm{S}} = \langle W, D, R, O, V \rangle$ where W is a set (of possible worlds); D is a set (of objects); $R \subseteq W \times W$ is reflexive, transitive, and symmetric; for $w \in W, O(w) \subseteq D$ is the set of objects existing in w; and for $A \in \text{TERM}, V(A) \subseteq D$ is the set of objects in the extension of a term A.

$V(A)$ is extended naturally to $V'(A, w) = V(A) \cap O(w)$. The truth conditions for the assertoric propositions are as expected:

Definition 6.2

$$\mathfrak{M}^{\mathrm{S}}, w \vDash AaB \quad \text{iff} \quad V'(B, w) \neq \emptyset \text{ and } V'(B, w) \subseteq V'(A, w).$$
$$\mathfrak{M}^{\mathrm{S}}, w \vDash AiB \quad \text{iff} \quad V'(A, w) \cap V'(B, w) \neq \emptyset.$$
$$\mathfrak{M}^{\mathrm{S}}, w \vDash AeB \quad \text{iff} \quad V'(A, w) \cap V'(B, w) = \emptyset.$$
$$\mathfrak{M}^{\mathrm{S}}, w \vDash AoB \quad \text{iff} \quad V'(B, w) = \emptyset \text{ or } V'(A, w) \nsubseteq V'(B, w).$$

The first conjunct ensures that our models satisfy existential import, which Aristotle accepted. Since R is an equivalence relation on W, the modalities L and M are the usual S5 modalities. What is novel in our semantics is the definition of the validity of a syllogism, which is given via the concept of model update:

Definition 6.3 For a model $\mathfrak{M}^{\mathrm{S}}$ and formula φ, the *update of* $\mathfrak{M}^{\mathrm{S}}$ *by* φ is the model $\mathfrak{M}^{\mathrm{S}} \restriction \varphi = \langle W \restriction \varphi, D, R \restriction \varphi, O \restriction \varphi, V \restriction \varphi \rangle$ where $W \restriction \varphi = \{w \in W : \mathfrak{M}^{\mathrm{S}}, w \vDash \varphi\}$; D is unchanged; and $R \restriction \varphi$, $O \restriction \varphi$, and $V \restriction \varphi$ are the restrictions of the original relations and functions to $W \restriction \varphi$.

Definition 6.4 A premise in a syllogism $\mathcal{S}$ is a *general rule* if (1) the middle term is distributed (cf. Def. 2.1 and §3) and (2) it is not particular.

A premise in a syllogism is a *special case* (1) if the other premise is a general rule and (2) only if it is not negative.

Definition 6.5 A syllogism $\mathcal{S}$ with special case s is valid for any simple model $\mathfrak{M}^{\mathrm{S}}$ and $w \in W$ iff (i) $\mathfrak{M}^{\mathrm{S}}, w \vDash M$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \vDash m$ imply (iii) $\mathfrak{M}^{\mathrm{S}} \restriction s, w \vDash c$.

The process of model update corresponds to the idea of "upgrading" the special case argued for by Rescher and McCall. When the general rule is necessary, we can consider only those worlds where the special case is in fact true, for if it is false then we do not care whether the conclusion is true or false, and when we restrict our attention in this fashion, we are able to draw necessary conclusions.

Note that on this definition, a syllogism $\mathcal{S}$ can be valid at world W in a simple model $\mathfrak{M}^{\mathrm{S}}$ even if the premises are true at w and the conclusion false. Thus, the definition of validity that we introduce is radically different from standard notions of validity, but this change in approach is justified by Aristotle's use syllogistics in scientific reasoning. If we either required that the conclusion already be true at w in $\mathfrak{M}^{\mathrm{S}}$, then we would collapse into the same problems that earlier attempts to formalize the modal syllogistic have, or defined validity so that syllogisms were only valid in $\mathcal{M}^{\mathrm{S}}$, then we would never have any valid modal syllogisms in the "real world".

As an example of how this system works, consider the problem of the two Barbaras. LXL Barbara is of the form $\mathcal{S} = \langle LAaB, BaC, LAaC \rangle$. To prove that this syllogism is

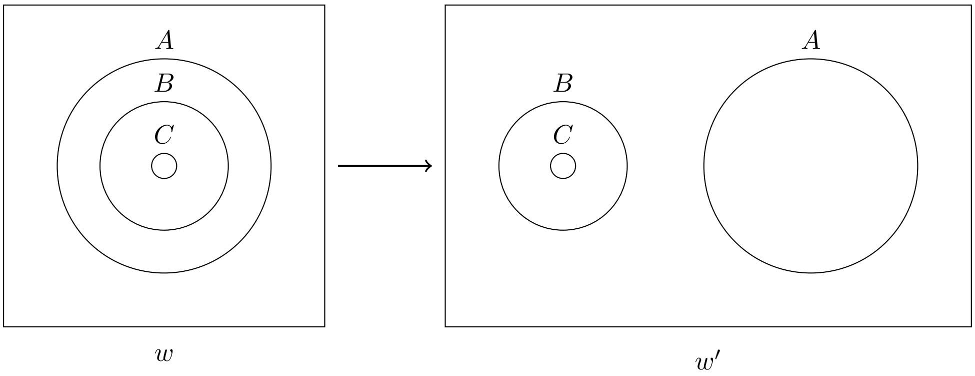

Figure 2. Countermodel for XLL Barbara.

valid, suppose that (i) $\mathfrak{M}^{\mathrm{S}}, w \models LAaB$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \models BaC$; we need to show that (iii) $\mathfrak{M}^{\mathrm{S}} \upharpoonright BaC, w \models LAaC$. By the definition of model restriction, $\mathfrak{M}^{\mathrm{S}} \upharpoonright BaC, w \models LBaC$ since the only worlds remaining are those worlds where BaC is true. From assumption (i), we also have that $\mathfrak{M}^{\mathrm{S}} \upharpoonright BaC, w \models LAaB$, and we show that LLL Barbara is valid in the next section. Our proof here is similar in spirit to Aristotle's, as he also reduces the case of LXL Barbara to LLL Barbara by using a type of conditional or relative necessity (cf. [9, §2]). In contrast, XLL Barbara ($= \langle AaB, LBaC, LAaC \rangle$) is not valid, as the counterexample in Figure 2 shows. It is straightforward to show that any syllogism derivable in McCall's L-X fragment [7, Table 7] is validated on these semantics and that any syllogism rejected has a countermodel on our semantics.

7 Adequacy and limitations of the semantics

In this section we prove the adequacy of the semantics introduced in the previous section. It is obvious that all of the purely assertoric syllogisms are valid on our semantics. Further, note that every valid second- or third-figure assertoric syllogism can be derived from one of the four perfect first-figure assertoric syllogism by means of simple conversion (from AeB infer BeA and vice versa, and from AiB infer BiA and vice versa), accidental conversion (from AaB infer AiB, and from AeB infer AoB), and *reductio ad absurdum* or contraposition (interchange the contradictory of the conclusion with either the contradictory of the major premise or the minor premise). In the assertoric syllogistic, contraposition is required only for the argument from XXX Barbara to XXX Bocardo and XXX Baroco. In the modal syllogistic, neither LXL nor XLL Bocardo or Baroco are valid; therefore, for the L-X fragment we do not need to consider proof by modal contraposition.

To prove the soundness of the semantics with respect to the L-X fragment of Aristotelian syllogistic, we first prove that the four perfect LLL syllogisms are valid:

Proposition 7.1 *LLL Barbara is valid on our semantics.*

Proof Take an arbitrary model $\mathfrak{M}^{\mathrm{S}}$ and assume that (i) $\mathfrak{M}^{\mathrm{S}}, w \models LAaB$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \models LBac$. It suffices to show that $\mathfrak{M}^{\mathrm{S}} \upharpoonright LBaC, w \models LAaC$. From the definition of model restriction, we have that $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, w \vDash LBaC$, which is equivalent to for all w', if wRw' then $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, w' \vDash BaC$, that is, $V'(C,w') \neq \emptyset$ and $V'(C,w') \subseteq V'(B,w')$. Further, from (i) we have w', if wRw' then $V'(B,w') \neq \emptyset$ and $V'(B,w') \subseteq V'(A,w')$. Now, take an arbitrary world v such that wRv, and we have that $V'(B,v) \neq \emptyset$, $V'(B,v) \subseteq V'(A,v)$, $V'(C,v) \neq \emptyset$, and finally $V'(C,v) \subseteq V'(B,v)$. Since subset inclusion is transitive, $V'(C,v) \subseteq V'(A,v)$, and hence $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, v \vDash AaC$. Since v was arbitrary, we have that this holds for all w' such that wRw', which is to $\mathfrak{M}^{\mathrm{S}} \restriction LBaA, w \vDash LAaC$. □

Proposition 7.2 *LLL Celarent is valid on our semantics.*

Proof Take an arbitrary model $\mathfrak{M}^{\mathrm{S}}$ and assume that (i) $\mathfrak{M}^{\mathrm{S}}, w \vDash LAeB$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \vDash LBaC$. Then it suffices to show that $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, w \vDash LAeC$. From the definition of restriction, we have that $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, w \vDash LBaC$, which is equivalent to for all w' if wRw' then $V'(C,w') \neq \emptyset$ and $V'(C,w') \subseteq V'(B,w')$. Further, from (i) we have for all w', if wRw' then $V'(A,w') \cap V'(B,w') = \emptyset$). Now, take an arbitrary world v such that wRv, then we have that (1) $V'(A,v) \cap V'(B,v) = \emptyset$) and that (2) $V'(C,v) \neq \emptyset$ and $V'(C,v) \subseteq V'(B,v)$. Now take an arbitrary $x \in D$ such that $x \in V'(C,v)$. By (2) we have $x \in V'(B,v)$ since $V'(C,v) \subseteq V'(B,v)$. Now, by (1) we have $x \notin V'(A,v)$. Since x was arbitrary, it follows that $V'(C,v) \cap V'(A,v) = \emptyset$. Hence $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, v \vDash AeC$. Since v was arbitrary, it follows that for any w' such that wRw', $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, w' \vDash AeC$ and so $\mathfrak{M}^{\mathrm{S}} \restriction LBaC, w \vDash LAeC$. □

Proposition 7.3 *LLL Darii is valid on our semantics.*

Proof Take an arbitrary model $\mathfrak{M}^{\mathrm{S}}$ and assume that (i) $\mathfrak{M}^{\mathrm{S}}, w \vDash LAaB$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \vDash LBiC$. It suffices to show that $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w \vDash LAiC$. From the definition of restriction, we have that $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w \vDash LBiC$, which is equivalent to for all w', if wRw' then $V'(B,w') \cap V'(C,w') \neq \emptyset$. Further, from (i) we have for all w', if wRw' then $V'(B,w') \neq \emptyset$ and $V'(B,w') \subseteq V'(A,w')$. Now, take an arbitrary world v such that wRv, and we have that (1) $V'(B,v) \neq \emptyset$ and $V'(B,v) \subseteq V'(A,v)$ and (2) $V'(B,v) \cap \ V'(C,v) \neq \emptyset$. Thus, we have that $\exists x \in V'(B,v) \cap V'(C,v)$. Call this element y, then since $y \in V'(B,v)$, it follows that $y \in V'(A,v)$. Since $y \in V'(C,v)$, then $V'(C,v) \cap V'(A,v) \neq \emptyset$, from which it follows that $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, v \vDash AiC$. Now since v was arbitrarily chosen, it follows that for all w' such that wRw', $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w' \vDash AiC$ and so $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w \vDash LAiC$. □

Proposition 7.4 *LLL Ferio is valid on our semantics.*

Proof Take an arbitrary model $\mathfrak{M}^{\mathrm{S}}$ be assume that (i) $\mathfrak{M}^{\mathrm{S}}, w \vDash LAeB$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \vDash LBiC$. It suffices to show that $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w \vDash LAoC$. From the definition of restriction, we have that $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w \vDash LAiC$. This and (i) are equivalent to for all w' such that wRw', (1) $V'(B,w') \cap V'(C,w') \neq \emptyset$ and (2) $V'(A,w') \cap V'(B,w') = \emptyset$. Now, take an arbitrary world v such that wRv. By (1), since $V'(B,v) \cap V'(C,v) \neq \emptyset$, it follows that $V'(C,v) \neq \emptyset$. Further, since $y \in V'(B,v)$ by (2) it follows that $y \notin V'(A,v)$. Hence, $V'(A,v) \nsubseteq V'(C,v)$, and thus $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w' \vDash AoC$. Now since v was arbitrary, it follows that for all w', wRw' implies that $\mathfrak{M}^{\mathrm{S}} \restriction LBiC, w' \vDash AoC$, as required. □

The soundness of the necessitated forms of simple and accidental conversion follow directly from their validity in their assertoric forms. The validity of the XLL and LXL syllogisms corresponds directly to the non-modalization of the premise other than one in which the middle term is distributed [5]; when we update with the special case, it becomes necessary, and thus the result corresponds to an LLL syllogism which can, if required, be converted to a first figure one.

However, when we attempt to extend these semantics to the L-X-M fragment, a number of problems emerge. First, we lose the close connection between the modality of the special case and the validity of the syllogism. We cannot retain the original definition of validity, since it makes valid a number of M-X syllogisms that would not have been accepted by Aristotle [6], for example MXM Barbara and MXM Celarent. This follows because when we update with the special case (the minor premise), $\mathfrak{M}^{\mathrm{S}} \restriction m, w \vDash m$ for all $w \in W$; then either the world which made the major premise true is still in the model, in which case the conclusion must also be true, or the conclusion is false in the updated model, but we have then falsified the major premise. This will be the case for any syllogism where the special case is non-modal; the update procedure will always promote an assertoric premise to a necessary one. A similar problem occurs when the special case is modal; since R is an equivalence relation, model reduction by a modal formula does not change the model, and thus we can always create a counter-model where the non-modalized form of the minor premise is true at a world other than the world where the major premises is true, and false elsewhere. Then the minor premise is possible, as required, but the conclusion is falsifiable.

The cause of both these problems is rooted in the same fact, namely, that our semantics does not preserve the validity of modal *reductio ad absurdum* (contraposition) when at least one premise is possible, rather than assertoric or necessary. While it may be possible to give a counterexample for every invalid syllogism in the L-X fragment, there is no straightforward way of converting this into a counterexample for the contraposed syllogism. The failure of contraposition in our semantics stems from the fact that there is no correlation between the premise that is modal and the premise that distributes the middle term in a syllogism and its contraposed form. For example, in MXM Camestres, the major premise is modalized, and the middle term is distributed in the minor; in its contraposition, XLL Ferison, the minor premise is modalized and the middle term is distributed in the major. On the other hand, in XMM Camestres, the minor is modalized and contains the distributed middle, whereas in its contraposition, XLL Darii, the minor premise remains modalized, but the middle term is distributed in the major premise. Thus, contraposition breaks the close association between modality and distribution of the middle that is seen in the L-X fragment of Aristotle's modal syllogistic. At this point, we have not seen a way to generalize our definition of validity

[5] The rather convoluted description "premise other than one in which the middle term is distributed" is a result of the fact that in Darapti, the middle term is distributed in *both* premises. Thus, either premise can serve as the general rule to the other's special case, which is reflected by the fact that XLL Darapti and LXL Darapti are both valid, and this is the only mood where both the XLL and LXL versions are.

[6] Aristotle doesn't explicitly discuss syllogisms with pure possibility (as opposed to contingency) premises; however, given his acceptance of proof by *reductio ad absurdum*, it is easy reconstruct which X-M syllogisms would have to be valid given the validity of the L-X fragment.

to validate contraposition, so that we can extend our results to the L-X-M fragment.

8 Conclusion

We have provided a semantics which validates the axiomatization for the L-X fragments of Aristotle's modal syllogistics proposed in [7]. These semantics, which crucially rely on a model update process, are much simpler than those found in previous literature, e.g., [3,4,6,15]. They take seriously Rescher's proposal in [8] that a modal syllogism should be interpreted as making a claim about a specific case of a general scientific principle. This emphasis on the status of the special case, the premise other than one where the middle term is distributed, gives rise to a type of relative or conditional necessity, which is expressed in our system by the model update process. This independent motivation for the use of the dynamic upgrade of the premise which is special case means that our system is not *ad hoc*, but instead has good philosophical grounding.

We have shown that a new definition of validity based on model update provides a sound semantics for the L-X fragment but we have also shown that it is not straightforward to extend this definition to a larger fragment. We hope to investigate such an extension in future work. Another question that we hope to answer in future work involves relating the semantics we gave for the L-X fragment to standard modern logical theories. Model updates such as the one that we have proposed, where the truth of a formula at the evaluating world is required before the update can proceed, correspond to truthful public announcements, à la the Public Announcement fragment of epistemic logic [16]. Thus, one natural open question is precisely what fragment of dynamic modal logic this fragment of Aristotle's syllogistic corresponds to.

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