Some Truths Are Best Left Unsaid

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Abstract

We study the formal properties of extensions of the basic public announcement logic by standard modal axioms such as D, T, 2, 4, and 5. We show that some of them fail to be conservative extensions of the underlying modal logic. This leads us to propose new truth conditions for announcements that better suit these extensions. The corresponding reduction axioms postulate the suitability of the updated model to the underlying logic. We show that if the fact can be expressed that the frame of an updated model is in the class of frames of the underlying modal logic, then the public announcement extension is axiomatisable. This is the case for, for example, K, KT, S4, and S4.3. We also show that such a formula does not exist for several logics whose frame condition involves existential quantification. This is the case for, for example, S4.2.

Keywords: public announcement logic, dynamic epistemic logic, reduction axioms.

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1 Introduction

In public announcement logics, one can write formulas like $\langle\varphi!\rangle\psi$ standing for "the announcement of φ can be made and, after that, ψ holds". Such logics have been extensively studied over the last years, starting with Plaza's paper [13] (reprinted in [14]); see [16] for an overview. In the literature, the term "Public Announcement Logic" is often used in the singular. But there is more than one such logic because the underlying modal logic of knowledge or belief may vary. In the literature, it is mainly the basic modal logic K and the modal logic $\mathsf{S5}$ that are investigated, or rather their multimodal versions. The reasons that are given for these choices are that K is an appropriate basis for a logic of belief, while $\mathsf{S5}$ is *the* logic of knowledge.

One may argue against $\mathsf{S5}$ as a logic of knowledge. It is typically taken for granted in artificial intelligence and game theory that it is $\mathsf{S5}$; however, this choice has been criticised in the philosophical literature, most prominently so by Lenzen [11,12]. The latter argues for $\mathsf{S4.2}$ and $\mathsf{S4.3}$ as the appropriate logics of knowledge. Let us remark that we only consider knowledge of a perfect reasoner, i.e. we leave the omniscience problem aside.

The base modal logic K is not a suitable logic of belief: philosophers insisted that such a logic should contain the D axiom since the seminal work of Hintikka [7]; Hintikka took KD, while later authors rather took $\mathsf{KD45}$ [11,12].

These considerations motivate a more systematic study of extensions of modal logics by public announcements. It turns out that many of these extensions are problematic. Semantically speaking, the problem is that the update of a model by an announcement may no longer be in the intended class of models: typically existential properties such as seriality may be lost after an update. Axiomatically speaking, as soon as we take it for granted that the public announcement operator is a normal modal operator, i.e. it obeys the K axiom and the necessitation rule, then the extensions of different modal logics may collapse.

In order to illustrate this let us show that the public announcement extension of KD coincides with the public announcement extension of KT if the announcement operator is normal. First, from the KD theorem $\neg\Box_i\bot$ we can infer

$$[\neg\varphi!]\neg\Box_i\bot$$

by the necessitation rule for announcements. Second, we have

$$\Box_i\varphi \to [\neg\varphi!]\Box_i\bot$$

by the usual reduction axioms for public announcements $[\neg\varphi!]\Box_i\bot \leftrightarrow (\neg\varphi \to \Box_i[\neg\varphi!]\bot)$ and $[\neg\varphi!]\bot \leftrightarrow \varphi$. From the above two and the fact that $[\neg\varphi!]$ is normal it follows that $\Box_i\varphi \to [\neg\varphi!]\bot$ is a theorem. The application of the reduction axioms uses the inference rule

$$\frac{\varphi \leftrightarrow \varphi'}{[\psi!]\varphi \leftrightarrow [\psi!]\varphi'}$$

which is derivable for normal modal operators. As $[\neg\varphi!]\bot$ reduces to φ we obtain that $\Box_i\varphi \rightarrow \varphi$ is a theorem. Another way of formulating this result is the following: if we extend KD by Plaza's reduction axioms (plus rules of replacement of equivalents for announcements) then one can prove the T axiom.

Given the above negative result we propose a new truth condition for public announcements that has not been studied before. It differs from the standard interpretation where the update is conditioned by the truth of the announcement: it requires moreover that the updated model is a *legal frame of the underlying logic*. We denote that interpretation by the superscript '$\mathcal{C}$' where $\mathcal{C}$ is a class of frames validating the underlying logic. We investigate the axiomatisability of the resulting public announcement logics. We give axiomatisations for all those of our logics where the fact that a frame is a $\mathcal{C}$ frame can be characterised in the language (more precisely, in its extension by the universal modal operator). On the negative side, the $\mathcal{C}$-semantics still does not allow to axiomatise a public announcement extension of $\mathsf{S4.2}$; the reason is that it cannot be characterised in the language of $\mathsf{S4.2}$ that a frame is a legal frame of the underlying logic $\mathcal{L}$.

The paper is organised as follows: In Section 2 we recall the standard presentation of public announcement logics: a semantics in terms of updates that are conditioned by the truth of the announcement and an axiomatics in terms of reduction axioms. In Section 3 we present our version of public announcement logics in terms of an enhanced truth condition. In Section 4 we characterise the validities of different classes of frames axiomatically by means of reduction axioms. In Section 5 we present several examples of classes of frames that still cannot be axiomatised. In Section 6 we discuss various other semantical options allowing to define variants of public announcement logics. In Section 7 we discuss some related work and in Section 8 we conclude.

2 Public announcement logics: standard version

Let $\mathbb{P}$ be a countable set of propositional letters and let $\mathbb{J}$ be a finite set of agent names. The public announcement language $\mathsf{L_{PAL}}$ is defined by the following BNF:

$$\varphi ::= p \mid \bot \mid \neg\varphi \mid (\varphi \vee \varphi) \mid \Diamond_i\varphi \mid \langle\varphi!\rangle\varphi$$

where p ranges over $\mathbb{P}$ and i ranges over $\mathbb{J}$. The length of φ, i.e. the number of occurrences of symbols in φ, will be denoted $\mathrm{l}(\varphi)$.

The epistemic language $\mathsf{L_{EL}}$ is the fragment of $\mathsf{L_{PAL}}$ without announcement operators $\langle\varphi!\rangle$. As usual, $\top$ abbreviates $\neg\bot$, $\Box_i\varphi$ abbreviates $\neg\Diamond_i\neg\varphi$, and $[\varphi!]\psi$ abbreviates $\neg\langle\varphi!\rangle\neg\psi$. We adopt the standard rules for omission of the parentheses.

2.1 Models and their updates

A *Kripke frame* is a tuple $\langle W, R\rangle$ such that:

- W is a nonempty set of possible worlds;

- $R : \mathbb{J} \to \wp(W \times W)$ associates to every agent $i \in \mathbb{J}$ a binary relation R_i on W.

The class of all Kripke frames is denoted by $\mathcal{C}_{\mathsf{all}}$.

A *Kripke model* is a tuple $M = \langle W, R, V \rangle$ such that $\langle W, R \rangle$ is a Kripke frame and $V : \mathbb{P} \to \wp(W)$ associates an interpretation $V(p) \subseteq W$ to each $p \in \mathbb{P}$. For every $x \in W$, the pair (M, x) is a *pointed model*. For convenience, we define $R_i(x) = \{x' \mid (x, x') \in R_i\}$. In an epistemic or doxastic interpretation, the elements of $R_i(x)$ are the worlds agent i considers possible at x: the worlds that are compatible with i's knowledge or respectively i's belief.

Let $M = \langle W, R, V \rangle$ be a Kripke model and let U be some subset of W. The *world update* (alias relativisation) of M by U is defined as $M \circ U = \langle W', R', V' \rangle$, with:

$$\begin{aligned} W' &= U \\ R'_i &= R_i \cap (U \times U), \quad \text{for every } i \in \mathbb{J} \\ V'(p) &= V(p) \cap U, \quad \text{for every } p \in \mathbb{P} \end{aligned}$$

If U is empty then $\langle W', R' \rangle$ is not a Kripke frame. We shall see that the standard truth condition avoids this by conditioning the update, preventing thus the semantics from being ill-defined.

If U is non-empty then $\langle W', R' \rangle$ is a Kripke frame. Things are less straightforward if we want to preserve membership in some subclass $\mathcal{C}$ of the class of all Kripke frames: it may for example happen that existential properties such as seriality, density, or confluence of the relations R_i are not always preserved. This will be the *raison d'être* of our enhanced semantics.

2.2 Standard truth conditions and validity

We recall the interpretation of $\mathsf{L}_{\mathsf{PAL}}$ formulas in a given model $M = \langle W, R, V \rangle$:

$$\begin{aligned} \|p\|_M &= V(p), \text{ for } p \in \mathbb{P} \\ \|\neg\varphi\|_M &= W \setminus \|\varphi\|_M \\ \|\varphi \vee \psi\|_M &= \|\varphi\|_M \cup \|\psi\|_M \\ \|\Diamond_i \varphi\|_M &= R_i^{-1}(\|\varphi\|_M) \\ &= \{x \in W \mid R_i(x) \cap \|\varphi\|_M \neq \emptyset\} \\ \|\langle \varphi! \rangle \psi\|_M &= \|\varphi\|_M \cap \|\psi\|_{M \circ \|\varphi\|_M} \end{aligned}$$

If we write $M, x \Vdash \varphi$ instead of $x \in \|\varphi\|_M$, then we can restate the last condition in a form that is perhaps more customary:

$$M, x \Vdash \langle \varphi! \rangle \psi \quad \text{iff} \quad M, x \Vdash \varphi \text{ and } M \circ \|\varphi\|_M, x \Vdash \psi$$

An $\mathsf{L}_{\mathsf{PAL}}$ formula φ is *globally true in the Kripke model* $M = \langle W, R, V \rangle$ if and only if $\|\varphi\|_M = W$. (This is sometimes noted $M \Vdash \varphi$.) We say that φ is *valid in the Kripke frame* $\langle W, R \rangle$ if and only if φ is globally true in every Kripke model over $\langle W, R \rangle$.

We are interested in several particular classes of Kripke frames: $\mathcal{C}_{\mathsf{all}}$ is the class of all Kripke frames; $\mathcal{C}_{\mathsf{serial}}$ is the class of frames where each accessibility relation is serial; $\mathcal{C}_{\mathsf{refl}}$ is the class of frames where each accessibility relation is reflexive; $\mathcal{C}_{\mathsf{confl}}$ is the class of frames where each accessibility relation is confluent; $\mathcal{C}_{\mathsf{refl,trans,eucl}}$ is the class of frames where each accessibility relation is an equivalence relation (reflexive, transitive and Euclidean); and so on.

Given a class of frames $\mathcal{C}$, let $\Lambda(\mathcal{C})$ be the set of $\mathsf{L_{PAL}}$ formulas that are valid in every frame of $\mathcal{C}$. An example of a formula that is valid in the class of all Kripke frames $\mathcal{C}_{\mathsf{all}}$ is $[p!]\Box_i p$, for atomic p. In contrast, the schema $[\varphi!]\Box_i\varphi$ is not valid in every Kripke frame. Another schema that is valid in every Kripke frame is $\Box_i\varphi \to [\neg\varphi!]\Box_i\bot$. It plays an important role in this paper: remember that we have already used it in the introduction when proving the T axiom.

Let us immediately say that the definition of $\Lambda(\mathcal{C})$ is problematic. Consider a Kripke model $M = \langle W, R, V\rangle$ such that R is serial (i.e., $\langle W, R\rangle \in \mathcal{C}_{\mathsf{serial}}$) and such that $M, x \Vdash \varphi \wedge \Box_i\neg\varphi$, and suppose we want to check whether $M, x \Vdash \langle\varphi!\rangle\Box_i\bot$. This involves checking whether $M \circ \|\varphi\|_M, x \Vdash \Box_i\bot$; which is the case, and therefore $M, x \Vdash \langle\varphi!\rangle\Box_i\bot$. This means that it may happen that after an announcement, agent i gets crazy and starts to believe everything. Formally speaking, while $\neg\Box_i\bot$ is valid in serial frames, its necessitation $[\varphi!]\neg\Box_i\bot$ is not: necessitation by announcements does not preserve validity! This is clearly undesirable. The key observation is that in $M \circ \|\varphi\|_M$ we have $R'_i(x) = \emptyset$: the accessibility relation R'_i is not serial any more.

The above discussion was about the preservation of seriality, but the same problem arises for example for confluence. More generally, it arises for classes of frames that are defined by an *existential condition.*

2.3 Axiomatisation of K-PAL

Let $\mathsf{K\text{-}PAL}$ be the least set of formulas in our language $\mathsf{L_{PAL}}$ that contains all instances of axiom schemas of the basic modal logic K for every $\Diamond_i$ and of the reduction axiom schemas

$$\langle\psi!\rangle p \leftrightarrow \psi \wedge p, \quad \text{for } p \in \mathbb{P} \qquad \mathsf{Red}_{\langle!\rangle,\mathbb{P}}$$
$$\langle\psi!\rangle\bot \leftrightarrow \bot \qquad \mathsf{Red}_{\langle!\rangle,\bot}$$
$$\langle\psi!\rangle\neg\varphi \leftrightarrow \psi \wedge \neg\langle\psi!\rangle\varphi \qquad \mathsf{Red}_{\langle!\rangle,\neg}$$
$$\langle\psi!\rangle(\varphi_1 \vee \varphi_2) \leftrightarrow \langle\psi!\rangle\varphi_1 \vee \langle\psi!\rangle\varphi_2 \qquad \mathsf{Red}_{\langle!\rangle,\vee}$$
$$\langle\psi!\rangle\Diamond_i\varphi \leftrightarrow \psi \wedge \Diamond_i\langle\psi!\rangle\varphi \qquad \mathsf{Red}_{\langle!\rangle,\Diamond_i}$$

and that is closed with respect to the inference rules of Modus Ponens, necessitation by announcements, and the two rules of equivalents for $\langle\psi!\rangle$:

$$\frac{\varphi \leftrightarrow \varphi'}{\langle\psi!\rangle\varphi \leftrightarrow \langle\psi!\rangle\varphi'} \qquad \mathsf{RE}^{\mathsf{r}}_{\langle!\rangle}$$
$$\frac{\psi \leftrightarrow \psi'}{\langle\psi!\rangle\varphi \leftrightarrow \langle\psi'!\rangle\varphi} \qquad \mathsf{RE}^{\mathsf{l}}_{\langle!\rangle}$$

Completeness of $\mathsf{K\text{-}PAL}$ w.r.t. $\mathcal{C}_{\mathsf{all}}$ follows from Wang's result that the axiom schemas of K for every $\Diamond_i$ plus the above reduction axioms plus the K axiom and the necessitation rule for announcements make up a complete axiomatisation of K-PAL [17, Corollary 1]: indeed, the K axiom and the necessitation rule for announcements may equivalently be replaced by $\mathsf{Red}_{\langle!\rangle,\bot}$, $\mathsf{Red}_{\langle!\rangle,\vee}$, and $\mathsf{RE}^{\mathsf{l}}_{\langle!\rangle}$ (cf. [4, Theorem 4.3]).

As customary in dynamic epistemic logics, our axiomatic system allows for a proof procedure in terms of reduction axioms. As there is no axiom schema for two consecutive announcements, reduction has to be performed 'bottom-up' (or 'inside-out' as Wang calls it [17]), starting by some innermost dynamic operator. The sound performance of such 'deep replacements' requires the rule of replacement of proved equivalents RRE; and indeed, our two rules of equivalents for announcements $\mathsf{RE}^{\mathsf{r}}_{\langle!\rangle}$ and $\mathsf{RE}^{\mathsf{l}}_{\langle!\rangle}$ enable the derivation of the rule

$$\frac{\chi \leftrightarrow \chi'}{(\varphi)^p_\chi \leftrightarrow (\varphi)^p_{\chi'}} \qquad \mathsf{RRE}$$

where $(\varphi)^p_\chi$ denotes the result of replacing all occurrences of p in φ by χ.

Proposition 2.1 *The rule* RRE *is derivable from the axiom schemas of* K *for* $\Diamond_i$ *by Modus Ponens, necessitation by announcements,* $\mathsf{RE}^{\mathsf{r}}_{\langle!\rangle}$*, and* $\mathsf{RE}^{\mathsf{l}}_{\langle!\rangle}$.

Proof. This follows from the fact that rules of equivalents can be derived for every (Boolean and modal) operator; remember that for the dynamic operators $\langle\psi!\rangle$ we directly have the two rules $\mathsf{RE}^{\mathsf{r}}_{\langle!\rangle}$ and $\mathsf{RE}^{\mathsf{l}}_{\langle!\rangle}$). For a proof see e.g. [4]. □

Here is an example of proof by means of reduction axioms:

$$\begin{aligned}
\langle p!\rangle\langle\neg q!\rangle r &\leftrightarrow \langle p!\rangle(\neg q \wedge r) && \mathsf{Red}_{\langle!\rangle,\mathbb{P}}\\
&\leftrightarrow \langle p!\rangle\neg q \wedge \langle p!\rangle r && \mathsf{Red}_{\langle!\rangle,\wedge}\\
&\leftrightarrow p \wedge \neg\langle p!\rangle q \wedge \langle p!\rangle r && \mathsf{Red}_{\langle!\rangle,\neg}\\
&\leftrightarrow p \wedge \neg(p \wedge q) \wedge p \wedge r && \mathsf{Red}_{\langle!\rangle,\mathbb{P}} \text{ (twice)}\\
&\leftrightarrow p \wedge \neg q \wedge r
\end{aligned}$$

In the second step, $\mathsf{Red}_{\langle!\rangle,\wedge}$ stands for the equivalence $\langle\psi!\rangle(\varphi_1 \wedge \varphi_2) \leftrightarrow \langle\psi!\rangle\varphi_1 \wedge \langle\psi!\rangle\varphi_2$ that can be proved from $\mathsf{Red}_{\langle!\rangle,\vee}$ and $\mathsf{Red}_{\langle!\rangle,\neg}$.

Observe that RRE is used in each of the steps. Observe also that we have to start by reducing the innermost dynamic operator $\langle\neg q!\rangle$ by means of the $\mathsf{Red}_{\langle!\rangle,\neg}$ rule —which requires the application of $\mathsf{RE}^{\mathsf{r}}_{\langle!\rangle}$— because our axiomatisation does not provide for a reduction axiom for the case of two consecutive dynamic operators.

2.4 Alternative axiomatisation

The axiomatisations in the literature typically lack $\mathsf{RE}^{\mathsf{r}}_{\langle!\rangle}$ and $\mathsf{RE}^{\mathsf{l}}_{\langle!\rangle}$. They instead have the following axiom schema for the composition of announcements:

$$\langle\psi_1!\rangle\langle\psi_2!\rangle\varphi \leftrightarrow \langle\langle\psi_1!\rangle\psi_2!\rangle\varphi \qquad \mathsf{Red}_{\langle!\rangle,\langle.!\rangle}$$

(see e.g. [13] or [16, Proposition 4.22]). To illustrate the difference we give a proof of the above example formula via $\mathsf{Red}_{\langle !\rangle,\langle .!\rangle}$.

$$\begin{aligned}
\langle p!\rangle\langle\neg q!\rangle r &\leftrightarrow \langle\langle p!\rangle\neg q!\rangle r && \mathsf{Red}_{\langle !\rangle,\langle .!\rangle}\\
&\leftrightarrow \langle p!\rangle\neg q \wedge r && \mathsf{Red}_{\langle !\rangle,\mathbb{P}}\\
&\leftrightarrow p \wedge \neg\langle p!\rangle q \wedge r && \mathsf{Red}_{\langle !\rangle,\neg}\\
&\leftrightarrow p \wedge \neg(p \wedge q) \wedge r && \mathsf{Red}_{\langle !\rangle,\mathbb{P}}\\
&\leftrightarrow p \wedge \neg q \wedge r
\end{aligned}$$

This is an 'outside-in' reduction, as opposed to reductions without $\mathsf{Red}_{\langle !\rangle,\langle .!\rangle}$ which have to proceed 'inside-out' and require our above rule of replacement of proved equivalents RRE [17].

RRE is stated in Proposition 4.46 in [16], and its proof (Exercise 4.48) says that it follows from the fact that the necessitation rule for announcements is an admissible inference rule (cf. also Exercise 4.52 there). We note in passing that Wang proved that neither RRE nor $\mathsf{RE}^{\mathsf{r}}_{\langle !\rangle}$ can be derived from the axiom schemas of K plus the above reduction axioms alone, i.e., without $\mathsf{RE}^{\mathsf{r}}_{\langle !\rangle}$, $\mathsf{RE}^{\mathsf{l}}_{\langle !\rangle}$, or $\mathsf{Red}_{\langle !\rangle,\langle .!\rangle}$ [17].

Remark 2.2 It follows from the completeness theorem (and from the fact that RRE preserves validity) that RRE is admissible, i.e., it preserves theoremhood: for every *formula instance* $\chi \leftrightarrow \chi'$ and every *formula instance* φ, if there is a proof of $\chi \leftrightarrow \chi'$ then there is a proof of $(\varphi)^p_\chi \leftrightarrow (\varphi)^p_{\chi'}$. However, this does not mean that RRE is derivable, i.e., that there is a derivation of the inference rule RRE itself. This situation can be compared to the completeness theorem for K-PAL which only says that every valid formula is provable, but does not guarantee that there are proofs of all valid formula schemas. These two versions of completeness —w.r.t. schemas and w.r.t. instances— coincide for logics where the rule of uniform substitution preserves validity, but K-PAL does not have that property. We note in passing that it is only recently that a complete axiomatisation of the schematic validities of K-PAL was given [8].

2.5 When things get wrong: public announcement extensions of KD, KD45, S4.2, etc.

We have just seen how K-PAL completely axiomatises the set of formulas that are valid in the class of all Kripke frames. Let KT-PAL denote the least set of formulas in our language that contains K-PAL and all instances of the T axiom. Then KT-PAL completely axiomatises the set of formulas $\Lambda(\mathcal{C}_{\mathsf{refl}})$, i.e., the set of formulas that are valid in the class of all reflexive Kripke frames. In the same way, S5-PAL —the extension of K-PAL by the axiom schemas T, 4, and 5— axiomatises $\Lambda(\mathcal{C}_{\mathsf{refl,trans,eucl}})$.

More generally, one might naively expect that if the validities of a class of frames $\mathcal{C}$ in the non-dynamic language $\mathsf{L_{EL}}$ can be axiomatised by some set of schemas and rules $\mathcal{AX}_{\mathcal{C}}$, then the validities of $\mathcal{C}$ in $\mathsf{L_{PAL}}$ can be axiomatised by the set of schemas and rules for K-PAL plus $\mathcal{AX}_{\mathcal{C}}$. The following argument shows what happens when we do this for the logic KD.

By $\mathsf{Red}_{\langle !\rangle,\neg}$, $\mathsf{Red}_{\langle !\rangle,\Diamond_i}$, and $\mathsf{Red}_{\langle !\rangle,\mathbb{P}}$, one can derive $p \wedge \neg\Diamond_i p \to \langle p!\rangle\neg\Diamond_i\top$. We have seen in Section 2.3 that the rule of necessitation for announcements is derivable in our axiomatics of K-PAL and therefore also in the —hypothetical— axiomatisation of KD-PAL. Thus, in KD, by the axiom $\Diamond_i\top$ and the necessitation by announcement, one can derive $[p!]\Diamond_i\top$. From $p \wedge \neg\Diamond_i p \to \langle p!\rangle\neg\Diamond_i\top$ and $[p!]\Diamond_i\top$, one can obviously derive $\Box_i p \to p$.

This makes that φ is a theorem of the latter system if and only if φ is a theorem of KT-PAL, which is clearly undesirable.

It seems that in the case of serial frames the only way to 'save' the standard truth condition is to abandon the rule of necessitation by announcements. While this is technically possible, the price to pay is that in many cases, the extension of an underlying modal logic by public announcements cannot be a conservative extension of that underlying logic. We think that this should better be avoided.

3 A new semantics

The preceding observation has motivated us to design a new semantics for the logics of public announcements.

Our results typically require a master modality, such as the universal modality. We therefore add the latter to our language: we define the language $\mathsf{L}_{\mathsf{PAL},\forall}$ by the following BNF:

$$\varphi ::= p \mid \bot \mid \neg\varphi \mid (\varphi \vee \varphi) \mid \Diamond_i\varphi \mid \langle\varphi!\rangle\varphi \mid \forall\varphi$$

The formula $\exists\varphi$ abbreviates $\neg\forall\neg\varphi$. The language $\mathsf{L}_{\mathsf{EL},\forall}$ is the fragment of $\mathsf{L}_{\mathsf{PAL},\forall}$ without dynamic operators.

The universal modality $\forall$ is interpreted as follows:

$$\|\forall\varphi\|_M = \begin{cases} W & \text{if } \|\varphi\|_M = W \\ \emptyset & \text{else} \end{cases}$$

3.1 A parametrised truth condition

Let $\mathcal{C}$ be a given class of Kripke frames. Let M be some model over some frame of $\mathcal{C}$. In order to distinguish our semantics from the standard semantics we write $\|\varphi\|^{\mathcal{C}}_M$ instead of $\|\varphi\|_M$.

The truth conditions for all but the dynamic operator take the same form as before. For the latter we define:

$$\|\langle\varphi!\rangle\psi\|^{\mathcal{C}}_M = \begin{cases} \emptyset & \text{if the frame of } M \circ \|\varphi\|^{\mathcal{C}}_M \text{ is not in } \mathcal{C} \\ \|\varphi\|^{\mathcal{C}}_M \cap \|\psi\|^{\mathcal{C}}_{M\circ\|\varphi\|^{\mathcal{C}}_M} & \text{otherwise} \end{cases}$$

If we write $M, x \Vdash^{\mathcal{C}} \varphi$ instead of $x \in \|\varphi\|^{\mathcal{C}}_M$, then we can restate this condition in a form that is perhaps more customary:

$$\begin{aligned} M, x \Vdash^{\mathcal{C}} \langle\varphi!\rangle\psi \quad \text{iff} \quad & \text{the frame of } M \circ \|\varphi\|^{\mathcal{C}}_M \text{ is in } \mathcal{C}, \\ & M, x \Vdash^{\mathcal{C}} \varphi \text{ and } M \circ \|\varphi\|^{\mathcal{C}}_M, x \Vdash^{\mathcal{C}} \psi \end{aligned}$$

Remark that the above truth condition for the dynamic operator makes it a normal modal operator, i.e. the dynamic operator obeys the K axiom and the necessitation rule.

Obviously, given a class $\mathcal{C}$ of Kripke frames, what interests us is the set $\Lambda(\mathcal{C})^{\mathcal{C}}$ of $\mathsf{L}_{\mathsf{PAL}}$ formulas that are valid in every Kripke frame of the class $\mathcal{C}$ under our $\mathcal{C}$ conditioned interpretation. Traditionally, if a class $\mathcal{C}'$ of frames is included in a class $\mathcal{C}''$, then every formula valid in $\mathcal{C}''$ is also valid in $\mathcal{C}'$. In our new semantics, seeing that the truth conditions are conditioned by classes of frames, validity with respect to $\mathcal{C}''$ and validity with respect to $\mathcal{C}'$ are not equivalent notions. For some classes $\mathcal{C}'$ and $\mathcal{C}''$, it might appear that $\mathcal{C}' \subseteq \mathcal{C}''$ and $\Lambda(\mathcal{C}'')^{\mathcal{C}''} \not\subseteq \Lambda(\mathcal{C}')^{\mathcal{C}'}$. To see an example, let $\mathcal{C}'$ be the set of all strict total orders with at least 3 points and $\mathcal{C}''$ be the set of all strict total orders with at least 2 points. Obviously, $\mathcal{C}' \subseteq \mathcal{C}''$. In order to show that $\Lambda(\mathcal{C}'')^{\mathcal{C}''} \not\subseteq \Lambda(\mathcal{C}')^{\mathcal{C}'}$, let us consider the formula $\varphi = p \wedge [p!]\bot \to \Box\neg p$. We claim the following:

(i) $\varphi \in \Lambda(\mathcal{C}'')^{\mathcal{C}''}$;

(ii) $\varphi \notin \Lambda(\mathcal{C}')^{\mathcal{C}'}$.

To demonstrate **(i)**, let $M = \langle W, R, V\rangle$ be some model based on a linear order $\langle W, R\rangle$ in $\mathcal{C}''$. Hence, W contains at least 2 points. Let $x \in W$ be such that $M, x \Vdash^{\mathcal{C}''} p \wedge [p!]\bot$. Thus, $x \in V(p)$ but $V(p)$ does not contain at least 2 points. Therefore, $V(p) = \{x\}$ and $M, x \Vdash^{\mathcal{C}''} \Box\neg p$. To demonstrate **(ii)**, let $M = \langle W, R, V\rangle$ be a model based on the linear order $\{0, 1, 2\}$ in $\mathcal{C}'$ such that $V(p) = \{0, 1\}$. Obviously, $M, 0 \Vdash^{\mathcal{C}'} p \wedge [p!]\bot$ and $M, 0 \nVdash^{\mathcal{C}'} \Box\neg p$.

In sections 4 and 5 we will explore this $\mathcal{C}$ conditioned interpretation. We focus on axiomatisability in terms of reduction axioms. In that perspective, the crucial point is whether we are able to characterise the condition "the frame of $M \circ \|\varphi\|^{\mathcal{C}}_{M}$ belongs to class $\mathcal{C}$" in the logical language. For the cases where this is possible, our characterisations require in general a 'master modality' such as the common knowledge operator or the universal modality [1] (i.e., they require the language $\mathsf{L}_{\mathsf{EL},\forall}$); the only exception is the class $\mathcal{C}^{1}_{\mathsf{tr}}$ where $\mathbb{J}$ is a singleton and where the frames have transitive accessibility relations: no master modality is needed in that case. An example of such a frame class characterising condition, relative to a given announced formula ψ, is $\forall(\psi \to \bigwedge_{i\in\mathbb{J}} \Diamond_i \psi)$. Not surprisingly, this is the characterising formula for the class of serial frames. Before we formally introduce that, we have to further prepare the theoretical ground.

3.2 Reduction axioms

We define the *announcement degree* of a $\mathsf{L}_{\mathsf{PAL},\forall}$ formula φ as follows:

$$
\begin{aligned}
\mathrm{d}(p) &= 0\\
\mathrm{d}(\bot) &= 0\\
\mathrm{d}(\neg\varphi) &= \mathrm{d}(\varphi)\\
\mathrm{d}(\varphi \vee \psi) &= \max(\mathrm{d}(\varphi), \mathrm{d}(\psi))\\
\mathrm{d}(\Diamond_i\varphi) &= \mathrm{d}(\varphi)\\
\mathrm{d}(\langle\varphi!\rangle\psi) &= \max(\mathrm{d}(\varphi), \mathrm{d}(\psi)) + 1\\
\mathrm{d}(\forall\varphi) &= \mathrm{d}(\varphi)
\end{aligned}
$$

For example, the announcement degree of both $\langle p!\rangle\Diamond_i\langle\neg q!\rangle\Diamond_j r$ and $\langle\langle p!\rangle\neg q!\rangle(\Diamond_j r \vee \langle p!\rangle\top)$ is 2.

Consider some class of Kripke frames $\mathcal{C}$. Suppose the fact that the frame of the updated model $M \circ \|\varphi\|_M^{\mathcal{C}}$ belongs to $\mathcal{C}$ can be characterised by an $\mathsf{L}_{\mathsf{PAL},\forall}$ formula $\mathrm{f}(\varphi)$ whose announcement degree is at most that of φ. We then obtain the following reduction axioms:

$$
\begin{aligned}
\langle\psi!\rangle p &\leftrightarrow \psi \wedge \mathrm{f}(\psi) \wedge p \quad \text{for } p \in \mathbb{P} && \mathsf{Red}^{\mathcal{C}}_{\langle!\rangle,\mathbb{P}}\\
\langle\psi!\rangle\bot &\leftrightarrow \bot && \mathsf{Red}^{\mathcal{C}}_{\langle!\rangle,\bot}\\
\langle\psi!\rangle\neg\varphi &\leftrightarrow \psi \wedge \mathrm{f}(\psi) \wedge \neg\langle\psi!\rangle\varphi && \mathsf{Red}^{\mathcal{C}}_{\langle!\rangle,\neg}\\
\langle\psi!\rangle(\varphi_1 \vee \varphi_2) &\leftrightarrow \langle\psi!\rangle\varphi_1 \vee \langle\psi!\rangle\varphi_2 && \mathsf{Red}^{\mathcal{C}}_{\langle!\rangle,\vee}\\
\langle\psi!\rangle\Diamond_i\varphi &\leftrightarrow \psi \wedge \mathrm{f}(\psi) \wedge \Diamond_i\langle\psi!\rangle\varphi && \mathsf{Red}^{\mathcal{C}}_{\langle!\rangle,\Diamond_i}\\
\langle\psi!\rangle\exists\varphi &\leftrightarrow \psi \wedge \mathrm{f}(\psi) \wedge \exists\langle\psi!\rangle\varphi && \mathsf{Red}^{\mathcal{C}}_{\langle!\rangle,\exists}
\end{aligned}
$$

Observe that the announcement degree of formulas does not increase from the left to the right due to our hypothesis that the announcement degree of $\mathrm{f}(\varphi)$ is at most that of φ; this would be violated e.g. if $\mathrm{f}(\varphi)$ was $\langle\varphi!\rangle\top$.

Let us associate to every formula ψ in $\mathsf{L}_{\mathsf{PAL},\forall}$, its measure $\mathrm{m}(\psi) = (\mathrm{d}(\psi), \mathrm{l}(\psi))$ in $\mathbb{N}_0 \times \mathbb{N}_0$, $\mathbb{N}_0$ denoting the set of all the non-negative integers, and $\mathrm{d}(\psi)$ and $\mathrm{l}(\psi)$ respectively denoting the announcement degree and the length of ψ. Let $\ll$ be the well-founded ordering on $\mathbb{N}_0 \times \mathbb{N}_0$ defined by $(m_1, n_1) \ll (m_2, n_2)$ iff either $m_1 < m_2$, or $m_1 = m_2$ and $n_1 < n_2$.

The above reduction axioms suggest us to consider a function $\tau : \mathsf{L}_{\mathsf{PAL},\forall} \longrightarrow$

$\mathsf{L}_{\mathsf{EL},\forall}$ defined by the following equations:

$$
\begin{aligned}
\tau(p) &= p \\
\tau(\bot) &= \bot \\
\tau(\neg\varphi) &= \neg\tau(\varphi) \\
\tau(\varphi \vee \psi) &= \tau(\varphi) \vee \tau(\psi) \\
\tau(\Diamond_i\varphi) &= \Diamond_i\tau(\varphi) \\
\tau(\langle\psi!\rangle p) &= \tau(\psi) \wedge \tau(f(\psi)) \wedge p \\
\tau(\langle\psi!\rangle\bot) &= \bot \\
\tau(\langle\psi!\rangle\neg\varphi) &= \tau(\psi) \wedge \tau(f(\psi)) \wedge \neg\tau(\langle\psi!\rangle\varphi) \\
\tau(\langle\psi!\rangle(\varphi_1 \vee \varphi_2)) &= \tau(\langle\psi!\rangle\varphi_1) \vee \tau(\langle\psi!\rangle\varphi_2) \\
\tau(\langle\psi!\rangle\Diamond_i\varphi) &= \tau(\psi) \wedge \tau(f(\psi)) \wedge \Diamond_i\tau(\langle\psi!\rangle\varphi) \\
\tau(\langle\psi!\rangle\exists\varphi) &= \tau(\psi) \wedge \tau(f(\psi)) \wedge \exists\tau(\langle\psi!\rangle\varphi) \\
\tau(\exists\varphi) &= \exists\tau(\varphi)
\end{aligned}
$$

These equations really define a function by $\ll$-induction from $\mathsf{L}_{\mathsf{PAL},\forall}$ to $\mathsf{L}_{\mathsf{EL},\forall}$, seeing that if $\tau(\psi)$ occurs on the right side of the equation defining $\tau(\varphi)$ then $\mathrm{m}(\psi) \ll \mathrm{m}(\varphi)$.

In other respect, remark that for every ψ in $\mathsf{L}_{\mathsf{PAL},\forall}$, the formula

$$\psi \leftrightarrow \tau(\psi)$$

is valid in $\mathcal{C}$. It follows that when applying τ, we can eliminate step by step every occurrence of a dynamic operator. Therefore we can prove completeness of the axiomatisation obtained by replacing the standard reduction axioms by the above ones in the very same way as the completeness of the standard axiomatisation for K-PAL. Moreover, the validity problem of $\mathsf{L}_{\mathsf{PAL},\forall}$ formulas in the class $\mathcal{C}$ is reducible to the validity problem of $\mathsf{L}_{\mathsf{EL},\forall}$ formulas in $\mathcal{C}$.

So it remains to find out for which classes $\mathcal{C}$ such a function exists. This is the same as looking for a function $\mathrm{f} : \mathsf{L}_{\mathsf{PAL},\forall} \longrightarrow \mathsf{L}_{\mathsf{PAL},\forall}$ such that for every ψ in $\mathsf{L}_{\mathsf{PAL},\forall}$, the formula

$$\langle\psi!\rangle\top \leftrightarrow \psi \wedge \mathrm{f}(\psi)$$

is valid in $\mathcal{C}$ and the announcement degree of $\mathrm{f}(\psi)$ is at most that of ψ (the latter ensuring that reduction terminates). We do so in the next two sections.

4 Positive results

For which classes of frames $\mathcal{C}$ can we express that the frame of the updated model belongs to $\mathcal{C}$ by means of a formula $\mathrm{f}(\psi)$ of the language $\mathsf{L}_{\mathsf{PAL},\forall}$? Clearly, when $\mathcal{C}$ is the class of all Kripke frames $\mathcal{C}_{\mathsf{all}}$ then $\mathrm{f}(\psi) = \top$. The same is the case when $\mathcal{C}$ is a class of frames that is defined by universal first-order conditions [5], such as reflexivity, transitivity, symmetry, Euclideanity, linearity,

[5] A first-order formula is universal if it is of the form $\forall x_1 \dots \forall x_n \varphi$ where φ is quantifier-free and where the variables of φ are among $x_1, \dots, x_n$.

or combinations thereof: these conditions are preserved under any update. This accounts for the public announcement extensions of modal logics such as KT, $\mathsf{K4}$, $\mathsf{KT4} = \mathsf{S4}$, KB, KTB, $\mathsf{KB4}$, $\mathsf{KT45} = \mathsf{S5}$ and $\mathsf{S4.3}$.

Things are less straightforward for frames defined by seriality and combinations of seriality with other conditions such as transitivity and Euclideanity. In this section we exhibit formulas f(.) for some of these cases, thus accounting in particular for the public announcement logics $\mathsf{KD\text{-}PAL}$ and $\mathsf{KD45\text{-}PAL}$.

4.1 Universal frame conditions

Let Φ be a universal first-order sentence over $\{R, =\}$. Let $\mathcal{C}_\Phi$ be the class of frames satisfying Φ. Then

$$\langle \psi ! \rangle \top \leftrightarrow \psi$$

is valid in $\mathcal{C}_\Phi$. Therefore, we can set $\mathrm{f}_\Phi(\psi) = \top$.

This covers in particular the case where $\mathcal{C}$ is the class of frames for $\mathsf{S4.3}$, i.e., the class of reflexive, transitive and linear frames.

4.2 Seriality

The equivalence

$$\langle \psi ! \rangle \top \leftrightarrow \psi \wedge \forall (\psi \rightarrow \bigwedge_{i \in \mathbb{J}} \Diamond_i \psi)$$

is valid in the class of all serial frames $\mathcal{C}_{\mathsf{serial}}$. We can therefore set $\mathrm{f}(\psi) = \forall(\psi \rightarrow \bigwedge_{i \in \mathbb{J}} \Diamond_i \psi)$.

We observe that there is no $\mathsf{L_{EL}}$ formula φ such that $\langle p! \rangle \top \leftrightarrow \varphi$ is valid in serial frames. Suppose such a formula exists. Let n be its modal degree. Consider the frame $\langle \mathbb{N}_0, R \rangle$ where $\mathbb{N}_0$ is the set of all the non-negative integers and $\langle x, y \rangle \in R$ iff $y = x + 1$. This is clearly a serial frame. We define two valuations V and V' on that frame by stipulating $V(p) = \mathbb{N}_0$, $V'(p) = \{0, \cdots, n\}$, and $V(q) = \emptyset$ for every $q \neq p$. We have $\langle \mathbb{N}_0, R, V \rangle, 0 \Vdash^{\mathcal{C}_{\mathsf{serial}}} \varphi$ iff $\langle \mathbb{N}_0, R, V' \rangle, 0 \Vdash^{\mathcal{C}_{\mathsf{serial}}} \varphi$ because n is the modal degree of φ. However, $\langle \mathbb{N}_0, R, V \rangle, 0 \Vdash^{\mathcal{C}_{\mathsf{serial}}} \langle p! \rangle \top$ while $\langle \mathbb{N}_0, R, V' \rangle, 0 \nVdash^{\mathcal{C}_{\mathsf{serial}}} \langle p! \rangle \top$; the latter is the case because the frame of the updated model $M \circ \|p\|_M^{\mathcal{C}_{\mathsf{serial}}}$ is not serial. We therefore have a contradiction.

4.3 Seriality and transitivity

As the reader can check, the equivalence

$$\langle \psi ! \rangle \top \leftrightarrow \psi \wedge \forall (\psi \rightarrow \bigwedge_{i \in \mathbb{J}} \Diamond_i \psi)$$

is also valid in the class of all serial and transitive frames $\mathcal{C}_{\mathsf{serial,trans}}$. We can simplify that equivalence when the set of agents $\mathbb{J}$ is a singleton, say $\{i\}$.

Consider the class $\mathcal{C}^g_{\mathsf{serial,trans}}$ of frames for $\mathsf{KD4}$ that are point-generated, i.e. the class of serial and transitive frames $\langle W, R \rangle$ with a world $x \in W$ such that $W = \{x\} \cup R_i(x)$. Then

$$\langle \psi ! \rangle \top \leftrightarrow \psi \wedge \Diamond_i \psi \wedge \Box_i (\psi \rightarrow \Diamond_i \psi)$$

is valid in $\mathcal{C}^g_{\mathsf{serial,trans}}$. We may therefore set $\mathrm{f}(\psi) = \Diamond_i \psi \wedge \Box_i (\psi \rightarrow \Diamond_i \psi)$.

Example 4.1 Let us check that for the logic $\mathsf{KD4\text{-}PAL}$ the formula

$$(p \wedge \neg\Diamond_i p) \to \langle p!\rangle\neg\Diamond_i\top$$

is not valid. For $\mathcal{C}_{\mathsf{serial,trans}}$ we have the condition $\mathrm{f}(p) = \forall(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p)$. By the reduction axioms of Section 3.2 we obtain:

$$\begin{aligned}
\langle p!\rangle\neg\Diamond_i\top &\leftrightarrow p \wedge \forall\left(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p\right) \wedge \neg\langle p!\rangle\Diamond_i\top && \mathsf{Red}^{\mathcal{C}}_{\langle !\rangle,\neg}\\
&\leftrightarrow p \wedge \forall\left(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p\right) \wedge\\
&\quad \neg(p \wedge \forall\left(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p\right) \wedge \Diamond_i\langle p!\rangle\top) && \mathsf{Red}^{\mathcal{C}}_{\langle !\rangle,\Diamond_i}\\
&\leftrightarrow p \wedge \forall\left(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p\right) \wedge \neg\Diamond_i\langle p!\rangle\top && \text{(propos. simplif.)}\\
&\leftrightarrow p \wedge \forall\left(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p\right) \wedge\\
&\quad \neg\Diamond_i\left(p \wedge \forall\left(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p\right)\right) && \mathsf{Red}^{\mathcal{C}}_{\langle !\rangle,\top}
\end{aligned}$$

where the reduction axiom used in the last step can be obtained from $\mathsf{Red}^{\mathcal{C}}_{\langle !\rangle,\neg}$ and $\mathsf{Red}^{\mathcal{C}}_{\langle !\rangle,\bot}$. The $\mathsf{L}_{\mathsf{PAL},\forall}$ formula

$$(p \wedge \neg\Diamond_i p) \to \left(p \wedge \forall(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p) \wedge \neg\Diamond_i(p \wedge \forall(p \to \bigwedge_{i\in\mathbb{J}} \Diamond_i p))\right)$$

is not valid in serial and transitive frames. (It is actually even invalid in $\mathcal{C}_{\mathsf{all}}$.) Therefore

$$(p \wedge \neg\Diamond_i p) \to \langle p!\rangle\neg\Diamond_i\top$$

is not valid either. This contrasts with the hypothetical proof system that we have discussed in Section 2.5.

4.4 Seriality, transitivity, and Euclideanity: single agent case

Let us suppose that the set of agents $\mathbb{J}$ is the singleton $\{i\}$. Consider the class $\mathcal{C}^{g}_{\mathsf{serial,trans,eucl}}$ of frames for $\mathsf{KD45}$ that are point-generated, i.e. the class of serial, transitive and Euclidean frames $\langle W, R\rangle$ with a world $x \in W$ such that $W = \{x\} \cup R_i(x)$.

Just as for serial frames the equivalence

$$\langle \psi! \rangle \top \leftrightarrow \psi \wedge \forall(\psi \to \bigwedge_{i \in \mathbb{J}} \Diamond_i \psi)$$

is valid in $\mathcal{C}^g_{\mathsf{serial,trans,eucl}}$. However, we can do better because there is only one agent: the schema

$$\langle \psi! \rangle \top \leftrightarrow \psi \wedge \Diamond_i \psi$$

is valid in $\mathcal{C}^g_{\mathsf{serial,trans,eucl}}$, i.e., $\mathrm{f}(\psi) = \Diamond_i \psi$.

We observe that the restriction to point-generated frames cannot be avoided if we want a characterisation by a $\mathsf{L_{EL}}$ formula: there is no $\mathsf{L_{EL}}$ formula φ such that

$$\langle p! \rangle \top \leftrightarrow \varphi$$

is valid in the class of serial, transitive and Euclidean frames. Indeed, suppose such a formula φ exists. Consider the frame $\langle W, R \rangle$ where $W = \{0, 1, 2\}$ and $R = \{\langle 0, 0 \rangle, \langle 1, 2 \rangle, \langle 2, 2 \rangle\}$. That frame is serial, transitive and Euclidean. Let $V_1(p) = \{0, 1\}$ and let $V_2(p) = \{0\}$. However, we have $\langle W, R, V_1 \rangle, 0 \not\Vdash^{\mathcal{C}_{\mathsf{serial,trans,eucl}}} \langle p! \rangle \top$, while $\langle W, R, V_2 \rangle, 0 \Vdash^{\mathcal{C}_{\mathsf{serial,trans,eucl}}} \langle p! \rangle \top$. Then we have $\langle W, R, V_1 \rangle, 0 \not\Vdash \varphi$ and $\langle W, R, V_2 \rangle, 0 \Vdash \varphi$. Since φ is without $\forall$ this leads us to a contradiction.

5 Negative results

We now show that there is no such formula as $\mathrm{f}(\psi)$ for the class of confluent frames, for the class of reflexive, transitive and confluent frames —i.e., for the basic logic $\mathsf{S4.2}$—, and for the class of dense frames.

5.1 Confluence

Let $\mathcal{C}_{\mathsf{confl}}$ be the class of all confluent frames. Is there a $\mathsf{L_{EL,\forall}}$ formula φ such that $\langle p! \rangle \top \leftrightarrow \varphi$ is valid in $\mathcal{C}_{\mathsf{confl}}$?

Suppose such a formula exists.

Let $W = \{x, y, z\}$ and let R_i be the reflexive and transitive closure of the relation $\{\langle x, y \rangle, \langle y, z \rangle\}$, for every i. The frame $\langle W, R \rangle$ is in $\mathcal{C}_{\mathsf{confl}}$. Let V be a valuation on $\langle W, R \rangle$ such that $V(p) = \{x, y\}$. The model $M = \langle W, R, V \rangle$ and its update by $\|p\|_M$ are depicted (without the reflexive edges) in Figure 1.

Fig. 1. M and its update by $\|p\|^{\mathcal{C}}_M$ (reflexive edges omitted)

Now let $W' = \{x, y, y', z\}$ and let R'_i be the reflexive and transitive closure of the relation $\{\langle x, y \rangle, \langle y, z \rangle, \langle x, y' \rangle, \langle y', z \rangle\}$, for every i. The frame $\langle W', R' \rangle$ is in $\mathcal{C}_{\mathsf{confl}}$, too. Let V' be a valuation on $\langle W', R' \rangle$ such that $V'(p) = \{x, y, y'\}$. The model $M' = \langle W', R', V' \rangle$ and its update by $\|p\|_{M'}$ are depicted (without the reflexive edges) in Figure 2.

The models M and M' are bisimilar [6], and we therefore have $M, x \Vdash \varphi$ iff

[6] The definition of bisimilarity has to take the universal modality into account. So $M =$

Fig. 2. M' and its update by $\|p\|^{\mathcal{C}}_{M'}$ (reflexive edges omitted)

$M', x \Vdash \varphi$ for every $\mathsf{L}_{\mathsf{EL},\forall}$ formula φ. However, we have $M, x \Vdash^{\mathcal{C}_{\mathsf{confl}}} \langle p! \rangle \top$, while $M', x \nVdash^{\mathcal{C}_{\mathsf{confl}}} \langle p! \rangle \top$; the former is the case because the frame of the updated model $M \circ \|p\|^{\mathcal{C}}_{M}$ is confluent, and the latter is the case because the frame of the updated model $M' \circ \|p\|^{\mathcal{C}}_{M}$ is not. We therefore have a contradiction.

5.2 Reflexivity, transitivity, and confluence

Let $\mathcal{C}_{\mathsf{refl,trans,confl}}$ be the class of all reflexive, transitive, and confluent frames. There is no $\mathsf{L}_{\mathsf{EL},\forall}$ formula φ such that $\langle p! \rangle \top \leftrightarrow \varphi$ is valid in that class. Indeed, suppose such a formula exists. We take over the two above counterexample models for confluence. Observe that both frames are in $\mathcal{C}_{\mathsf{refl,trans,confl}}$. Again, M and M' are bisimilar, and therefore $M, x \Vdash \varphi$ iff $M', x \Vdash \varphi$ for every φ. However, $M, x \Vdash^{\mathcal{C}_{\mathsf{refl,trans,confl}}} \langle p! \rangle \top$, while $M', x \nVdash^{\mathcal{C}_{\mathsf{refl,trans,confl}}} \langle p! \rangle \top$. We therefore have a contradiction.

5.3 Density

Let $\mathcal{C}_{\mathsf{dense}}$ be the class of all dense frames. There is no $\mathsf{L}_{\mathsf{EL},\forall}$ formula φ such that $\langle p! \rangle \top \leftrightarrow \varphi$ is valid in $\mathcal{C}_{\mathsf{dense}}$. Indeed, suppose such a formula exists. Let $\langle W, R \rangle$ be the frame defined by $W = \{\alpha, \omega, 1, 2, 3, 4, 5\}$ and

$$\begin{aligned} R_i = {} & \{\langle \alpha, \omega \rangle\} \cup \\ & \{\langle \alpha, y \rangle \mid 1 \leq y \leq 5\} \cup \\ & \{\langle x, \omega \rangle \mid 1 \leq x \leq 5\} \cup \\ & \{\langle x, y \rangle \mid 1 \leq x, y \leq 5, x \neq y\} \end{aligned}$$

for every $i \in \mathbb{J}$. The reader may check that $\langle W, R \rangle$ is indeed dense. Let $V_1(p) = \{\alpha, \omega, 1, 5\}$ and let $V_2(p) = \{\alpha, \omega, 1, 3, 5\}$. The models $M_1 = \langle W, R, V_1 \rangle$ and $M_2 = \langle W, R, V_2 \rangle$ are bisimilar, and therefore $M_1, \alpha \Vdash \varphi$ iff $M_2, \alpha \Vdash \varphi$ for every $\mathsf{L}_{\mathsf{EL},\forall}$ formula φ. However, $M_1, \alpha \nVdash^{\mathcal{C}_{\mathsf{dense}}} \langle p! \rangle \top$, while $M_2, \alpha \Vdash^{\mathcal{C}_{\mathsf{dense}}} \langle p! \rangle \top$. We therefore have a contradiction.

$\langle W, R, V \rangle$ and $M' = \langle W', R', V' \rangle$ are bisimilar if there is a relation $Z \subseteq W \times W$ such that:

(i) if $(x, x') \in Z$ then $x \in V(p)$ iff $x' \in V'(p)$, for every $p \in \mathbb{P}$

(ii) if $(x, x') \in Z$ and $(x, y) \in R_i$ then there is $y' \in W'$ such that $(x', y') \in R'_i$ and $(x, y) \in Z$

(iii) if $(x, x') \in Z$ and $(x', y') \in R'_i$ then there is $y \in W$ such that $(x, y) \in R_i$ and $(x, y) \in Z$

(iv) for every $x \in W$ there is $x' \in W'$ such that $(x, x') \in Z$

(v) for every $x' \in W'$ there is $x \in W$ such that $(x, x') \in Z$

The last two conditions say that both Z and its converse Z^{-1} are serial.

6 Discussion: semantic alternatives

We now explore some alternative semantics for public announcements that one can find in the literature: first, a proposal to update models in a different way, and second, a different formulation of the truth condition for public announcements.

6.1 Relation updates

Beyond the standard way of updating a Kripke model by eliminating *worlds* of Section 2, there are proposals in the literature where instead of worlds it is *edges* that are eliminated [5,6]. Let $M = \langle W, R, V \rangle$ be a model, and let U be some subset of W. The *relation update* of M by U is defined as $M \overset{\mathrm{r}}{\circ} U = \langle W, R', V \rangle$, with:

$$R'_i = R_i \cap (W \times U)$$

Let $\Lambda(\mathcal{C})^{\mathrm{r}}$ be the set of $\mathsf{L_{PAL}}$ formulas valid in $\mathcal{C}$ under relation update (under our truth condition for announcements of Section 2.2 requiring truth of the announcement).

We argue that if one wants the underlying modal logic to be a customary logic of knowledge or belief, then this way of extending a modal logic by announcements is not very interesting, for two reasons. First, as far as the language $\mathsf{L_{PAL}}$ is concerned we have $\Lambda(\mathcal{C}) = \Lambda(\mathcal{C})^{\mathrm{r}}$ (because the generated submodels of $M \overset{\mathrm{r}}{\circ} U$ and $M \circ U$ are equal); the logics differ only when the universal modal operator comes into play. Second, while membership in the class of all models $\mathcal{C}_{\mathsf{all}}$ is preserved under relation update, membership in a particular class of models is preserved in fewer cases: not only do existential first-order conditions such as seriality, density and confluence fail, but also universal conditions such as reflexivity and symmetry.

6.2 An unconditioned truth condition

Remember that the standard formulation of the truth condition requires announcements to be *truthful.* This means that the agents acquire knowledge. In the literature one can find not only another definition of model update, but also another formulation of the truth condition for public announcements. To the contrary, in Gerbrandy's formulation announcements may be false [5,6]. The latter formulation is therefore often claimed to be more appropriate for agents acquiring beliefs, see e.g. [9]. We call this the *unconditioned* truthcondition and highlight it by "u":

$$M, x \Vdash^{\mathrm{u}} \langle \varphi ! \rangle \psi \quad \text{iff} \quad M \circ \|\varphi\|^{\mathrm{u}}_M, x \Vdash \psi$$

Then we have two options, according to whether we use world update or relation update. Call $\Lambda(\mathcal{C})^{\mathrm{u}}$ and $\Lambda(\mathcal{C})^{\mathrm{ru}}$ the resulting logics of the class of frames $\mathcal{C}$. For example, Kooi's basic public announcement logic is $\Lambda(\mathcal{C}_{\mathsf{all}})^{\mathrm{ru}}$ [9].

Observe that none of the logics $\Lambda(\mathcal{C})^{\mathrm{u}}$ makes sense. Indeed, consider the case where a model M is updated by a formula that is false at every point of

M; then the set of possible worlds of the updated model is empty, and therefore the update of the model is not a legal Kripke model: the unconditioned interpretation is ill-defined. Let us finally notice that when we 'repair' the logics $\Lambda(\mathcal{C})^{\mathrm{u}}$ by replacing the unconditioned truth condition by our enhanced version then we obtain $\Lambda(\mathcal{C})^{\mathcal{C}}$.

7 Related work

Yanjing Wang has recently investigated axiomatisations of public announcement logics [17]. He focusses on public announcement extensions of the basic modal logic K and subtleties with different versions of the axiomatization. In particular, he highlights the role of the rule of replacement of equivalents $\mathsf{RE}^{\mathsf{r}}_{\langle !\rangle}$. His work certainly provide a stimulating background to our own. Updates that preserve $\mathsf{KD45}$ have been investigated in [15,2,10]. Guillaume Aucher [2] defines a language fragment that makes you go mad ('crazy formulas'). The formula characterising the cases where this can be avoided is the same as ours in Section 4. David Steiner [15] proposes that the agent does not incorporate the new information if he already believes to the contrary. In that case, nothing happens. Otherwise, access to states where the information is not believed is eliminated, just as for believed public announcements. This solution to model unbelievable information is similarly proposed in the elegant [10], where it is called 'cautious update' — a suitable term. The difference between these approaches and ours is that the agent simply keeps his old beliefs in case the new information is unbelievable (i.e., if there is no accessible state where the announced formula is true). In our KD45 preserving updates the update cannot be executed if it is unbelievable.

8 Conclusion

In this paper we had a closer look at the axiomatization and the semantics of various public announcement logics. We highlighted problems that arise for epistemic or doxastic logics with existential frame conditions such as seriality or confluence and proposed an enhanced truth condition avoiding these problems in some cases. Our new truth condition amounts to the original condition if the basic logic is K or $\mathsf{S5}$. We have studied the limitations of our solution; in particular the case of confluence remains without a satisfactory solution, and with it the extension of the logic of knowledge $\mathsf{S4.2}$ by public announcements.

Our results required to extend the language by a master modality. We opted for the universal modality; however, the common knowledge modality would do, too.

Everything said here transfers to other kinds of updates such as assignments. More precisely, in dynamic epistemic logics with assignments, we can model announcements that stay within a certain frame class in the same way, but for the dynamics involving assignments there are no additional complications: an assignment is a total function that can always be executed, and that never changes the frame properties of the transformed model. It would be interesting to study whether (and how) it transfers to dynamic epistemic logics with event models and product update [3]. In [3], Baltag and Moss showed that reflexivity, transitivity and Euclideanity are preserved under standard product updates. In [2], Aucher provided a characterisation of the condition f(ψ) under which product update preserves seriality.

Here is a proposal for a way to overcome the expressive limitations that we have highlighted in Section 5. The idea is to enrich the language by a modal constant $\delta_{\mathcal{C}}$ whose interpretation is that it is true exactly when the frame it is evaluated in is part of the class $\mathcal{C}$. Let us call that language $\mathsf{L}_{\mathsf{PAL},\delta_{\mathcal{C}}}$. Its truth condition is:

$$\langle W, R, V\rangle, x \Vdash \delta_{\mathcal{C}} \text{ iff } \langle W, R\rangle \in \mathcal{C}$$

Let us define a translation from $\mathsf{L}_{\mathsf{PAL}}$ to $\mathsf{L}_{\mathsf{PAL},\delta_{\mathcal{C}}}$ whose main clauses are:

$$p^{\mathsf{t}} = p$$
$$(\langle\psi!\rangle\varphi)^{\mathsf{t}} = \langle\psi^{\mathsf{t}}!\rangle(\delta_{\mathcal{C}} \wedge \varphi^{\mathsf{t}})$$

and homomorphic for the other cases. We then have that for every frame $\langle W, R\rangle$, every valuation V over that frame, and every world $x \in W$, $\langle W, R, V\rangle, x \Vdash^{\mathcal{C}} \varphi$ iff $\langle W, R, V\rangle, x \Vdash \varphi^{\mathsf{t}}$. It remains however to axiomatise the $\mathcal{C}$ validities in the augmented language.

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