

Foundational Issues in Logical Dynamics

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A healthy field combines two driving forces, extension of coverage and theoretical reflection – and modal logic is no exception. In this talk, we look at some broader theoretical issues raised by recent developments in dynamic logics of information flow, whose main technical feature is the modal study of definable model change. Our pilot system will be public announcement logic PAL, the base calculus of update with hard information. We identify several general themes behind the particulars of its design. These include model-theoretic issues like the balance of expressive power for the static and dynamic components of the language, and learnability results in the form of preservation theorems. But we also discuss PAL's calculus of reasoning, via correspondence analysis of its axioms as constraints on update functions in domains of inquiry, showing how this leads to two recent extensions: protocol versions where no reduction to the static base language occurs, and substitution-closed core versions of the system. Both moves exemplify more general operations on modal logics.

Next, we consider other dynamic-epistemic logics, and amplify on the above themes, while adding some new ones. First, we discuss event models and product update in DEL, a calculus of model construction that shows analogies with process algebra, but also with the epistemic μ -calculus. Then we turn to dynamic logics for belief change under incoming hard and soft information that upgrade plausibility relations without eliminating worlds. We discuss what logical format best captures the generality needed to avoid a jungle of revision policies. In this setting also, we look at the general phenomenon of defining new static modalities by dynamic considerations.

While all this broadens our view of single-step model transformations, another conspicuous dimension in current research is the global temporal horizon of a process of inquiry, a conversation, or a game, adding procedural information to local updates. We discuss representation results linking local dynamic and global temporal logics, while also pointing at specific model-theoretic issues arising with concrete processes involving iterated hard or soft announcements, and their limit behavior. Here we link up with modal fixed-point logics, though the general situation is far from clear. As a very concrete instance of merging local and global perspectives, we discuss some recent links between dynamic epistemic logics and formal learning theory, where modal models acquire a new meaning as priors pre-encoding learning methods.

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Drawing these threads together, we will discuss what it means to be a dynamic logic, looking for postulates on model transformations at the level of abstract model theory.

This survey by no means exhausts the foundational themes behind current dynamic logics of information and agency. We refer to the cited literature for further issues of logic combination and entanglement of modal operators in areas of agency such as games, with new questions about expressiveness and computational complexity for modal languages capturing reasoning about best action in interactive social settings. Likewise, recent work has questioned the use of standard relational models, using more finely-grained neighborhood models generalizing many of the above concerns. A final general theme are interfaces between modal and probabilistic approaches, which are emerging naturally across many of the areas discussed in this lecture.

Our presentation will refer to earlier literature plus talks at AiML 2012, but its main sources are J. van Benthem, 1996, *Exploring Logical Dynamics*, CSLI Publications, Stanford, 2011, *Logical Dynamics of Information and Interaction*, Cambridge University Press, Cambridge, 2012, *Logic in Games*, The MIT Press, Cambridge Mass.